United States Department of Agriculture
Soil Conservation Service
Engineering Division

Technical Release No. 15
Design Section
January 31, 1961

COMPUTATION OF WATER SURFACE PROFILES AND RELATED PARAMETERS BY ES-CHARTS

The objectives of this Technical Release are to present (1) a description of the charts ES-125 through ES-134, and (2) procedure for the determination of water surface profiles and related parameters by the ES-charts.

The theory, definitions, and nomenclature used in this Technical Release are presented in the National Engineering Handbook, Section 5, Hydraulics.

Mr. Paul D. Doubt, Head, Design Section, developed the IBM 650 program used to compute the parameters required to plot the ES-charts. Plotting the ES-charts and writing this Technical Release was done by the members of the Design Section under his direct supervision.

The members of the Design Section who contributed to this Technical Release are: Messrs. Arthur R. Gregory, Gerald E. Oman and Norman P. Hill, Civil Engineers; Richard C. Meininger and Francis M. Wysong, Engineering Aids; and Mrs. Dorothy A. Stewart, Secretary.

TECHNICAL RELEASE

NUMBER 15

COMPUTATION OF WATER SURFACE PROFILE AND RELATED PARAMETERS BY ES-CHARTS

CHAPTER 1 - DESCRIPTION OF ES-CHARTS

The theory, definitions, and nomenclature used in this Technical Release are presented in the National Engineering Handbook, Section 5, Hydraulics.

The procedure for water surface profile determination is a modification of Escoffier's[1] method for natural channels. The modification includes the effect of changes in velocity head on the profile, and the charts are a slightly different plot than used in Escoffier's original presentation.

The procedure presented is useful in the design of prismatic channels. It readily shows the effect on the profile of changes in the tailwater condition, bottom slope, Manning's roughness coefficient, and bottom width of the channel. These charts are useful in determining the profiles associated with various steady discharges. They are not applicable for unsteady flow conditions.

The charts are prepared for rectangular, trapezoidal, and circular prismatic channels. A catalog of the various charts available is given on page 1-2. Generally, six sheets are used to present the parameters for each bottom width of a channel with a particular side slope z:1. A lap of about one-third of the depth increment is shown from one sheet to the next. The sheets are so arranged that the curves on one side of any sheet do not lap the curves on its reverse side.

Numerous combinations of grid sizes were used to permit a reasonable degree of accuracy of the graphical solution. The values given by the charts were obtained entirely by the use of an electronic digital computer. Data were computed at depth intervals equal to or smaller than the scale graduations of the chart. The original drawings of charts were twice the linear size of the multilithed prints.

Each sheet has two independent line charts (see page 1-3). The vertical line chart on the left side of the sheet is composed of the two scales $Q_{c,d}$ and d. The d-scale is the same as the scale on the axis of ordinates of the central chart.

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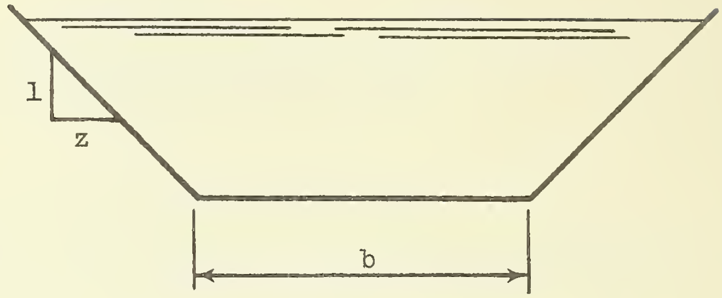
[1] Escoffier, Francis F., Graphical Calculation of Backwater Eliminates Solution by Trial, Engineering News-Record, June 27, 1946.

CATALOG OF ES-CHARTS
FOR COMPUTATION OF WATER SURFACE PROFILES

z	ES-Number
* 0:1	125
*** $\frac{1}{2}$:1	126
* 1:1	127
* $1\frac{1}{2}$:1	128
* 2:1	129
* $2\frac{1}{2}$:1	130
* 3:1	131
*** 4:1	132
*** 6:1	133
** Circular	134

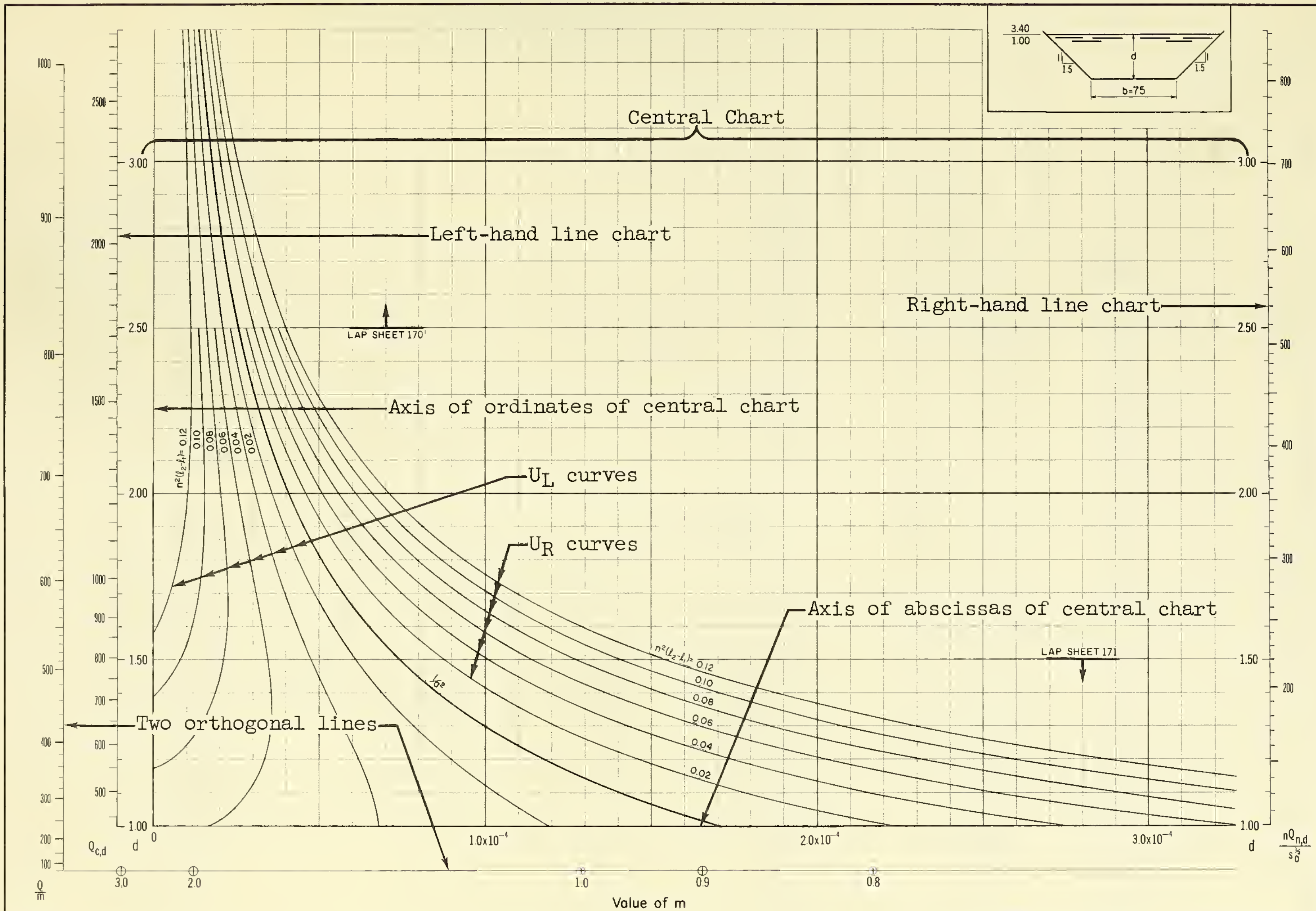


b	Sheet Numbers	Depth Range	b	Sheet Numbers	Depth Range
1	1 - 6	0.2 - 10.0	70	163 - 168	0.2 - 30.0
2	7 - 12	0.2 - 10.0	75	169 - 174	0.2 - 30.0
3	13 - 18	0.2 - 10.0	80	175 - 180	0.2 - 30.0
4	19 - 24	0.2 - 10.0	85	181 - 186	0.2 - 30.0
5	25 - 30	0.2 - 10.0	90	187 - 192	0.2 - 30.0
6	31 - 36	0.2 - 15.0	95	193 - 198	0.2 - 30.0
7	37 - 42	0.2 - 15.0	100	199 - 204	0.2 - 30.0
8	43 - 48	0.2 - 15.0	110	205 - 210	0.2 - 30.0
9	49 - 54	0.2 - 15.0	120	211 - 216	0.2 - 30.0
10	55 - 60	0.2 - 15.0	130	217 - 222	0.2 - 30.0
12	61 - 66	0.2 - 20.0	140	223 - 228	0.2 - 30.0
14	67 - 72	0.2 - 20.0	150	229 - 234	0.2 - 30.0
16	73 - 78	0.2 - 20.0	160	235 - 240	0.2 - 30.0
18	79 - 84	0.2 - 20.0	170	241 - 246	0.2 - 30.0
20	85 - 90	0.2 - 20.0	180	247 - 252	0.2 - 30.0
22	91 - 96	0.2 - 25.0	190	253 - 258	0.2 - 30.0
24	97 - 102	0.2 - 25.0	200	259 - 264	0.2 - 30.0
26	103 - 108	0.2 - 25.0	210	265 - 270	0.2 - 30.0
28	109 - 114	0.2 - 25.0	220	271 - 276	0.2 - 30.0
30	115 - 120	0.2 - 25.0	230	277 - 282	0.2 - 30.0
35	121 - 126	0.2 - 30.0	240	283 - 288	0.2 - 30.0
40	127 - 132	0.2 - 30.0	250	289 - 294	0.2 - 30.0
45	133 - 138	0.2 - 30.0	260	295 - 300	0.2 - 30.0
50	139 - 144	0.2 - 30.0	270	301 - 306	0.2 - 30.0
55	145 - 150	0.2 - 30.0	280	307 - 312	0.2 - 30.0
60	151 - 156	0.2 - 30.0	290	313 - 318	0.2 - 30.0
65	157 - 162	0.2 - 30.0	300	319 - 324	0.2 - 30.0

* Available for $b \leqq 75$. Available for $80 \leqq b \leqq 300$ by special request. Delivery of special request orders is usually slower than available stock material.

** Available after January 1962

*** Not to be drawn by Design Section. A limited supply of computations ($1 \leqq b \leqq 300$) is available for plotting charts.



REFERENCE
These Water Surface Profile Charts were developed and computed by Paul D. Daubt, head, Design Section.

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN SECTION

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The vertical line chart located on the right-hand side of the sheet is composed of the two scales $\left[\frac{nQ_{n,d}}{s_o^{1/2}}\right]$ and d. This d-scale is also the same as the scale given on the axis of ordinates of the central chart.

These line charts may be used to determine various hydraulic parameters. The determination of some of these parameters will require a mental redesignation of the scales of the line charts. This will not be difficult if the relations of the various parameters are well understood. Such redesignations are given under Chapter 2 "Determination of Hydraulic Parameters".

The use of the central chart and the two orthogonal lines whose scales have the labeled values m and $\frac{Q}{m}$ will be discussed later.

CHAPTER 2 - DETERMINATION OF HYDRAULIC PARAMETERS FROM LINE CHARTS

The procedures for determining various hydraulic parameters are given in the following numbered paragraphs. The examples 1-10 on pages 4-1 through 4-5 are numbered to correspond with the numbered procedures given here.

Procedure 1.

The critical discharge $Q_{c,d}$ of a given channel corresponding to a depth d is read directly from the left-hand line chart.

Procedure 2.

The critical depth $d_{c,Q}$ of a given channel corresponding to a discharge Q is read directly from the left-hand line chart. When the discharge Q is given, the parameters of this line chart are mentally redesignated as Q and $d_{c,Q}$.

Procedure 3.

The normal discharge $Q_{n,d}$ of a given channel corresponding to a depth d is determined by the use of the right-hand line chart.

a. For the depth d read the corresponding value of $\left[\frac{nQ_{n,d}}{s_o^{1/2}}\right]$.

b. Compute the value of $\left[\frac{n}{s_o^{1/2}}\right]$ for the given channel, or find the value of $\left[\frac{n}{s_o^{1/2}}\right]$ from ES-138.

c. Divide the value of $\left[\frac{nQ_{n,d}}{s_o^{1/2}}\right]$ read from the line chart by the value obtained for $\left[\frac{n}{s_o^{1/2}}\right]$.

Procedure 4.

The normal depth $d_{n,Q}$ of a given channel corresponding to a discharge Q is determined from the right-hand line chart. When the value of Q is given and the value of $d_{n,Q}$ is to be determined, the parameters of this chart are mentally redesignated as $\left[\frac{nQ}{s_o^{1/2}}\right]$ and $d_{n,Q}$.

a. Compute the value of $\left[\frac{n}{s_o^{1/2}}\right]$ for the given channel or find the value of $\left[\frac{n}{s_o^{1/2}}\right]$ from ES-138.

b. Compute $\left[\frac{nQ}{s_o^{1/2}}\right] = \left[\frac{n}{s_o^{1/2}}\right] Q$.

c. For the computed value of $\left[\frac{nQ}{s_o^{1/2}}\right]$, read the corresponding normal depth $d_{n,Q}$.

Procedure 5.

The normal slope $s_{n,Qd}$ of a given channel corresponding to the discharge Q and depth d is also determinable from the right-hand line chart. When the values of Q and d are given and the corresponding normal slope $s_{n,Qd}$ is to be determined, the parameters of the line chart are mentally redesignated as

$\left[\frac{nQ}{(s_{n,Qd})^{1/2}}\right]$ and d.

a. For the depth d read the corresponding value of $\left[\frac{nQ}{(s_{n,Qd})^{1/2}}\right]$.

b. Method 1. Compute nQ, and divide by the value of $\left[\frac{nQ}{(s_{n,Qd})^{1/2}}\right]$. Square the result to obtain $s_{n,Qd}$.

That is,

$$s_{n,Qd} = \left[\frac{nQ}{\left[\frac{nQ}{(s_{n,Qd})^{1/2}}\right]}\right]^2$$

Method 2. Divide the value of $\left[\frac{nQ}{(s_{n,Qd})^{1/2}}\right]$ by Q to determine $\left[\frac{n}{(s_{n,Qd})^{1/2}}\right]$. Mentally redesignate the parameters $\left(s_o,\ n, \text{ and } \left[\frac{n}{s_o^{1/2}}\right]\right)$ of ES-138 as $\left(s_{n,Qd},\ n, \text{ and } \left[\frac{n}{(s_{n,Qd})^{1/2}}\right]\right)$.

For the given n and determined $\left[\frac{n}{(s_{n,Qd})^{1/2}}\right]$, read $s_{n,Qd}$.

Procedure 6.

The critical slope $s_{c,d}$ of a given channel corresponding to the depth d is determined by the use of both the right- and left-hand line charts. The parameters of the right-hand line chart are mentally redesignated as $\left[\frac{nQ_{c,d}}{(s_{c,d})^{1/2}}\right]$ and d when the critical slope corresponding to a depth is to be determined.

a. Read the values of $Q_{c,d}$ and $\left[\frac{nQ_{c,d}}{(s_{c,d})^{1/2}}\right]$ corresponding to the depth d from the two line charts.

b. Method 1. Compute $nQ_{c,d}$, and divide by the value of $\left[\frac{nQ_{c,d}}{(s_{c,d})^{1/2}}\right]$. Square this result to obtain $s_{c,d}$.

That is

$$s_{c,d} = \left[\frac{nQ_{c,d}}{\left[\frac{nQ_{c,d}}{(s_{c,d})^{1/2}}\right]}\right]^2$$

Method 2. Divide the value of $\left[\frac{nQ_{c,d}}{(s_{c,d})^{1/2}}\right]$ by $Q_{c,d}$ to obtain the value of $\left[\frac{n}{(s_{c,d})^{1/2}}\right]$. Mentally redesignate the parameters $\left(s_o, n, \text{ and } \left[\frac{n}{s_o^{1/2}}\right]\right)$ of ES-138 as $\left(s_{c,d}, n, \text{ and } \left[\frac{n}{(s_{c,d})^{1/2}}\right]\right)$. Read the value of $s_{c,d}$ from ES-138. For the given n and determined $\left[\frac{n}{(s_{c,d})^{1/2}}\right]$, read $s_{c,d}$.

Procedure 7.

The critical slope $s_{c,Q}$ of a given channel corresponding to the discharge Q is determined by the use of both the right- and left-hand line charts. The parameters of the left-hand line chart are mentally redesignated as Q and $d_{c,Q}$, and the parameters of the right-hand line chart are mentally redesignated as $\left[\dfrac{nQ}{(s_{c,Q})^{1/2}}\right]$ and $d_{n,Q}$.

a. For the discharge Q read the corresponding value of $d_{c,Q}$. By the definition of critical slope observe that $d_{c,Q} = d_{n,Q}$.

b. For the value of $d_{c,Q} = d_{n,Q}$, read the corresponding value of $\left[\dfrac{nQ}{(s_{c,Q})^{1/2}}\right]$ from the right-hand line chart.

c. Method 1. Compute nQ, and divide by the value of $\left[\dfrac{nQ}{(s_{c,Q})^{1/2}}\right]$, and square the result to obtain the critical slope.

That is

$$s_{c,Q} = \left[\frac{nQ}{\left[\dfrac{nQ}{(s_{c,Q})^{1/2}}\right]}\right]^2$$

Method 2. Divide the values of $\left[\dfrac{nQ}{(s_{c,Q})^{1/2}}\right]$ by Q to determine $\left[\dfrac{n}{(s_{c,Q})^{1/2}}\right]$.

Mentally redesignate the parameters $\left(s_o,\ n,\ \text{and} \left[\dfrac{n}{{s_o}^{1/2}}\right]\right)$ of ES-138 as $\left(s_{c,Q},\ n,\ \text{and} \left[\dfrac{n}{(s_{c,Q})^{1/2}}\right]\right)$. For the given n and determined $\left[\dfrac{n}{(s_{c,Q})^{1/2}}\right]$, read $s_{c,Q}$.

<u>Procedure 8.</u>

<u>The rate of friction loss $s \equiv \dfrac{dh_f}{d\ell}$ corresponding to the discharge Q and depth d</u> is determined by the use of the right-hand line chart. Theory shows this rate is

$$s \equiv \frac{dh_f}{d\ell} = s_o \left[\frac{Q}{Q_{n,d}}\right]^2$$

where s is dimensionless or has the units foot pound per foot length of channel per pound of water.

a. For the depth d read the corresponding value of $\left[\frac{nQ_{n,d}}{s_o^{1/2}}\right]$.

b. Divide $\left[\frac{nQ_{n,d}}{s_o^{1/2}}\right]$ by n, and square the result to obtain $\left[\frac{Q^2_{n,d}}{s_o}\right]$.

c. Divide Q^2 by $\left[\frac{Q^2_{n,d}}{s_o}\right]$ to obtain $s = s_o \left[\frac{Q}{Q_{n,d}}\right]^2$.

Procedure 9.

The direction of the solution line is required before water surface profiles can be determined. It is also used to determine velocity head, specific energy head, and the total friction loss between the two sections used to determine a depth of flow. The direction of the solution line, for a given sheet, depends only on the discharge Q.

The direction of the solution line is determined from the two orthogonal lines found along the bottom and extreme left-hand side of the sheet. Each of these orthogonal lines have a single scale. (See page 1-3.) The value of m selected must be used in determining the value of $\frac{Q}{m}$. On any one sheet and for a given value of Q, the direction of the solution line is the same for any selected value of m.

The following procedure is used to determine the direction of the solution line.

a. Select the smallest value of m for which the corresponding value of $\frac{Q}{m}$ can be read on the $\frac{Q}{m}$ scale.

b. The direction of the solution line corresponding to the discharge Q is the same as that vector drawn from the point m to the point $\frac{Q}{m}$.

The tangent of the angle which the solution line makes with the axis of abscissas is $\left[\frac{Q^2}{2g}\right]$. The direction of the solution line corresponding to the discharge $Q_{c,d}$ is parallel to the tangent of the $\frac{1}{a^2}$ curve at the depth d.

Procedure 10.

The converse situation of determining the discharge Q corresponding to a given direction of the solution line frequently arises.

a. Select the smallest value of m on the m-scale such that a line having the given direction will intersect the $\frac{Q}{m}$ scale when drawn from the point m.

b. Read the value of $\frac{Q}{m}$ at the intersection, and multiply by the selected value of m to obtain the corresponding value of discharge Q.

CHAPTER 3 - DETERMINATION OF SPECIFIC ENERGY HEAD, VELOCITY HEAD, AND WATER SURFACE PROFILES.

The method of determining various parameters and the water surface profiles by use of the central chart are given by the following numbered procedures. The examples on pages 4-5 through 4-17 illustrating the use of the procedures are numbered to correspond with the numbered procedures given here.

Procedure 11.

The specific energy head is the vertical distance between the energy grade line and the bottom of the channel. Its value is often used in hydraulic calculations. The graphical solution for the specific energy head $H_{e,Qd}$ of a given channel corresponding to the depth d and discharge Q is made by constructing a line having the direction of the solution line from the $\frac{1}{a^2}$ curve at depth d to intersect the axis of ordinates. The value read at the intersection of the axis of ordinates is the specific energy head $H_{e,Qd}$.

$$H_{e,Qd} = d + \left[\frac{v^2}{2g}\right] \equiv d + \left[\frac{Q^2}{2ga^2}\right]$$

The proof of this construction is readily seen by observing that the tangent of the angle formed by the solution line with the horizontal is $\left[\frac{Q^2}{2g}\right]$ and the value of the abscissa of the $\frac{1}{a^2}$ curve at d is $\frac{1}{a^2}$. Therefore, $\left[\frac{Q^2}{2g}\right] \cdot \frac{1}{a^2} = \left[\frac{v^2}{2g}\right]$.

Procedure 12.

Generally, when the specific energy head $H_{e,Q}$ and the discharge Q are given, two possible corresponding depths can be ascertained. The graphical determination of the two depths corresponding to a given specific energy head $H_{e,Q}$ and discharge Q is given.

a. From the value of the specific energy head read on the axis of ordinates, draw a line having the direction of the solution line to intersect the $\frac{1}{a^2}$ curve (usually two points).

b. Read the corresponding depths at the intersections of the $\frac{1}{a^2}$ curve.

Procedure 13.

The corresponding discharge Q can be graphically determined when the specific energy head $H_{e,d}$ and depth d are given.

Determine the discharge Q corresponding to the direction of the line drawn from the value $H_{e,d}$ on the axis of ordinates to the point on the $\frac{1}{a^2}$ curve at depth d. (See procedure 10).

Procedure 14.

The unique situation of determining the critical discharge and critical depth corresponding to a given specific energy head frequently arises in hydraulic computations. An example of the need for such a computation is the determination of the discharge for a reservoir through a control section into a channel. A direct solution for the critical discharge and depth is made graphically by the use of ES-charts.

The graphical solution for the critical discharge Q_c and depth d_c corresponding to the specific energy head H_e is made by mentally redesignating the axis of ordinates as values of specific energy head and following the procedure below.

a. From the value of the specific energy head on the axis of ordinates, draw a line tangent to the $\frac{1}{a^2}$ curve.

b. Determine the discharge Q_c corresponding to the direction of the solution line parallel to the tangent. (See procedure 10.)

c. Read the value of d_c corresponding to the discharge Q_c determined in b. (See procedure 2).

Though it is noted that the solution line drawn in step a is tangent to the $\frac{1}{a^2}$ curve at depth d_c, the point of tangency is not well defined. Solution by the given procedure is the most accurate.

Procedure 15.

The graphical solution for the velocity head $\left[\frac{v^2}{2g}\right]$ of a given channel corresponding to the depth d and discharge Q is made by determining the specific energy head (see procedure 11) and subtracting the depth d.

Procedure 16.

The determination of a critical slope corresponding to a given Manning's coefficient n and discharge Q (or depth d) is always unique. The determination of the discharge Q (or depth d) for which a given s_o is the critical slope may have no solution, one solution, or two solutions.

The determination of the discharges Q (or depths d) for which the given bottom slope s_o is critical is, arithmetically, a trial-and-error solution. This procedure gives a graphical solution for any possible solutions for either discharge Q or depth d within the depth range of the charts.

a. Arbitrarily select values of depth d for the depth range of the chart. Prepare a tabulation of d, $Q_{c,d}$, $Q_{n,d}$ and $[Q_{n,d} - Q_{c,d}]$.

b. Plot values of $\left([Q_{n,d} - Q_{c,d}] \text{ vs } d\right)$.

c. Read the solutions of depths d for which the given bottom slope s_o is critical at $[Q_{n,d} - Q_{c,d}] = 0$ from the chart prepared in step b. The discharges Q for which s_o is a critical slope are $Q_{c,d}$.

Water Surface Profile in a Computational Reach

Water surface profiles for a given discharge Q are frequently required in the work of the Soil Conservation Service. They are required in drainage, irrigation, flood prevention, and other conservation works to determine depths of flow. Water surface profiles are used:

1. to ascertain the capacity of a channel,
2. to ascertain tailwater depth on a hydraulic structure,
3. to ascertain actual tractive forces on the bottom and sides of a channel,
4. to ascertain the discharge in a delivery channel,
5. to locate a hydraulic jump, and
6. for many other objectives.

The water surface profile is determined for a steady discharge Q. Thus, for every steady discharge there is a corresponding water surface profile. Further, a water surface profile depends on a starting depth. A starting depth is the depth of flow which is given or pre-determined before profile computations are started. It is emphasized again that the water surface profile depends on the discharge and the starting depth.

The determination of a water surface profile is merely the determination of the depth of flow at selected channel sections when the channel is conveying the discharge Q. Two consecutive selected sections form a computational reach. When the starting depth of flow is known (or has been determined) at one of these two sections the depth is determined at the other section. This determined depth then becomes a known depth, which is used as a starting depth for the next computational reach, and the depth is determined at the next selected section. This is repeated for each computational reach until the depth has been determined at every selected section.

The method of determining the water surface profile in a channel when the starting depth is not known is given by Procedure 18, which requires a complete understanding of Procedure 17.

Procedure 17 is used to determine the unknown depth at the end of the computational reach when the starting depth is known. The determination of the unknown depth involves an integration. Since it is not possible to formally integrate the general differential equation of varied flow, the solution is often obtained by numerical integration. The numerical integration is an approximation that may be made as accurate as desired by selecting computational reaches "sufficiently short". The charts ES-125 through ES-134 are designed to give a graphical solution for this integration. The graphical solution may be made as accurate as the charts can be read, assuming the computational reach $(\ell_2-\ell_1)$ is "sufficiently short". Whether or not a reach is "sufficiently short" depends on the degree of accuracy desired by the user. The process of determining the depth by the use of the chart also furnishes the friction losses to indicate the degree of accuracy of the depth determination. This will be considered after the explanation of the depth determination.

The graphical determination of a depth of flow at the end of a "sufficiently short reach" requires the following data to be given:

1. the steady discharge Q - cfs,
2. a starting depth d - ft,
3. cross-sectional shape and size,
4. Manning's coefficient n,
5. bottom slope of the channel s_o - ft/ft, and
6. the tolerable error to be permitted in the depth determination.

Procedure 17.

The procedure for determining the depth when the starting depth is known is given on sheet 3 of ES-142. It assumes the user selects the value of $n^2(\ell_2-\ell_1)$ which in turn fixes the value of the length of the computational reach $(\ell_2-\ell_1)$ i.e., the distance between the stations containing the starting depth and the depth to be determined.

When the starting depth is less than $d_{c,Q}$ flow is super-critical and computations must be carried in a downstream direction. In other words, the section containing the depth to be determined is downstream a distance of $(\ell_2-\ell_1)$ from the section containing the starting depth. When the starting depth is greater than $d_{c,Q}$ flow is subcritical and computations must be carried in an upstream direction.

Drawing the solution line to determine the depth at the end of the computational reach also furnishes the change in velocity head and the friction loss in the reach. See sheet 1 of ES-142 which illustrates the determination of velocity heads and friction losses when flow is subcritical, supernormal, accelerated, and on a positive sloped channel. The graphical integration of energy loss by friction is an approximation. It depends on the depth of flow which generally varies throughout the reach of the channel. The rate of energy loss between the sections of the starting depth and the section of the determined depth is taken as the arithmetical average of the instantaneous rate of friction loss of the two end sections of the computational reach (see procedure 8). This is equivalent to saying that the friction loss for half of the computational reach is evaluated with a depth of flow equal to the starting depth, and the friction loss for the other half of the computational reach is evaluated with a depth of flow equal to the determined depth of flow.

The error in this approximation is always less than the difference of these two evaluated friction losses $|h_{f_2} - h_{f_1}|$, since the depth varies monotonically.

The instantaneous rate of friction loss for the depth of flow d is

$$\frac{dh_f}{d\ell} = s_o \left[\frac{Q}{Q_{n,d}}\right]^2 .$$

The friction loss for the distance $\left[\frac{(\ell_2-\ell_1)}{2}\right]$ is

$$h_f = s_o \left[\frac{Q}{Q_{n,d}}\right]^2 \left[\frac{(\ell_2-\ell_1)}{2}\right] \equiv \left[\frac{Q^2}{2g}\right] \left[\frac{s_o}{Q^2_{n,d}}\, g(\ell_2-\ell_1)\right] .$$

The distance between the $\frac{1}{a^2}$ -curve and the U_R-curve (or U_L-curve) is $\left[\frac{s_o}{Q^2_{n,d}}\, g(\ell_2-\ell_1)\right]$ at the depth d, and the tangent of the angle formed by the solution line with the horizontal is $\left[\frac{Q^2}{2g}\right]$. The friction loss for a depth d over the distance $\left[\frac{(\ell_2-\ell_1)}{2}\right]$ is the vertical component of the projection on the solution line of the horizontal line segment between the $\frac{1}{a^2}$ -curve and the U_R-curve (or the $\frac{1}{a^2}$ -curve and the U_L-curve).

The energy relations of varied flow shown on sheet 1 of ES-142 is for one case of the twelve types of water surface curves in prismatic channels. Sheet 2 of ES-142 summarizes the typical graphical procedures for depth determination for all cases of varied flow. Observe that for subcritical flow the starting point A is on the U_R-curves, and for supercritical flow the starting point A is on the U_L-curves.

Water Surface Profiles in a System of Channels

The water surface profile is often desired for a system of channels which may contain hydraulic structures and pools. The solution for profiles for such a system of channels resolves into one of three problems, that is:

1. The determination of the water surface profile corresponding to a given steady discharge Q and a tailwater depth when flow is subcritical, or the determination of a water surface profile corresponding to a given discharge Q and headwater depth when flow is supercritical.

2. The determination of the steady discharge Q in a channel for a headwater depth and a tailwater depth corresponding to the unknown discharge Q. This is often a trial-and-error problem which involves only the principles of the first stated problem for selected discharges. Examples of problems which have a given headwater and tailwater depth instead of a given discharge are:

    a. When flow is subcritical in a bifurcated channel a headwater condition is imposed at the section of forking. The headwater condition at the bifurcated section is usually assumed to be level. This fact is used to confirm the headwater depths and determine the divided discharges, which also depend on the tailwater depth. This is a water surface profile problem.

    b. A delivery channel from a pool has a given headwater elevation at the outlet of the pool. The elevation of the water surface in the pool along with the tailwater depth will determine the discharge in the channel. When the outlet of the pool is not a control section, this becomes a water surface profile problem. It was tacitly assumed no outlet structure exists at the entrance of the channel in this example.

3. The determination of a tailwater depth for a given discharge Q and headwater depth when flow is subcritical, or the determination of a headwater depth for a given discharge Q and tailwater depth when flow is supercritical. For prismatic channels having no break in grade, this solution can be obtained by one water surface profile determination.

Procedure 18.

For profile determination, a channel containing hydraulic structures and/or pools is divided into reaches which do not contain a structure nor a pool. These reaches are called channels under consideration and are considered individually and successively in an upstream direction. A channel under consideration may or may not have a structure or pool at its end section. The water surface profile depends on the discharge and starting depth. Since structures and pools at the ends of a channel under consideration can affect the discharge and starting depth, these effects are evaluated before commencing water surface profile computations. A brief description of the effects of pools and the various types of structures on discharge and starting depths are considered.

The evaluation of the pool level will determine the discharge Q in the channel conveying or delivering water from the pool. The determination of the discharge Q from the pool is simple when the entrance to the channel is known to be a control section. For this situation the pool depth at the entrance of the delivery is $H_{e,Q}$. When the entrance section is not a control section, a rating curve is computed for the entrance section. This requires that water surface profiles corresponding to various discharges be computed upstream to the pool.

The overfall type of hydraulic structure in a channel, whether located at the outlet of a pool or not, often is a control section at which the depth of flow at the weir is known to be critical depth. If the elevation of the tailwater on the overfall type of structure is greater than the elevation of the critical depth over the weir, the weir section is not a control section; and the water surface profile corresponding to various discharges are computed upstream to the weir section. A rating curve for the submerged weir section is computed from these profiles.

Sluice structures in a channel usually control the discharge Q and the starting depth at the upstream end of a channel under consideration. When they do not control the starting depth, the tailwater on the sluice must be computed to determine the discharge. Again, this is accomplished by computing water surface profiles to obtain the tailwater on the sluice. The headwater on the sluice can then be determined by its hydraulic characteristics for the given discharge.

Closed conduits may or may not flow full. Water surface profiles may need to be computed in partially filled closed conduits to determine rating curves. Technical Release No. 14 gives the method of obtaining the rating curve for rectangular and circular culverts with all possible tailwater conditions by the use of a digital computer. When the closed conduit is flowing full and the tailwater has no control on the discharge, the discharge is determined entirely by the hydraulic characteristics of the structure. When the tailwater does have an effect on the discharge, then the water surface profile computations are required to obtain the discharge of the structure.

When pools and hydraulic structures solely control the discharge, the profile for the channels under consideration connecting the pool or structure is determined independently. However, it is not always possible to ascertain before the profile computations are made whether or not the pool or structure solely controls the discharge.

The same given data is required for water surface profile computations in each of the channels under consideration as is required for computational reaches.

This data is:

1. the steady discharge Q - cfs,
2. a starting depth d - ft,
3. cross-sectional shape and size,
4. Manning's coefficient n,
5. bottom slope of the channel, and
6. the tolerable error to be permitted in the depth determination.

Often the discharge Q is not given or the starting depth is not given. The following are the three possible cases.

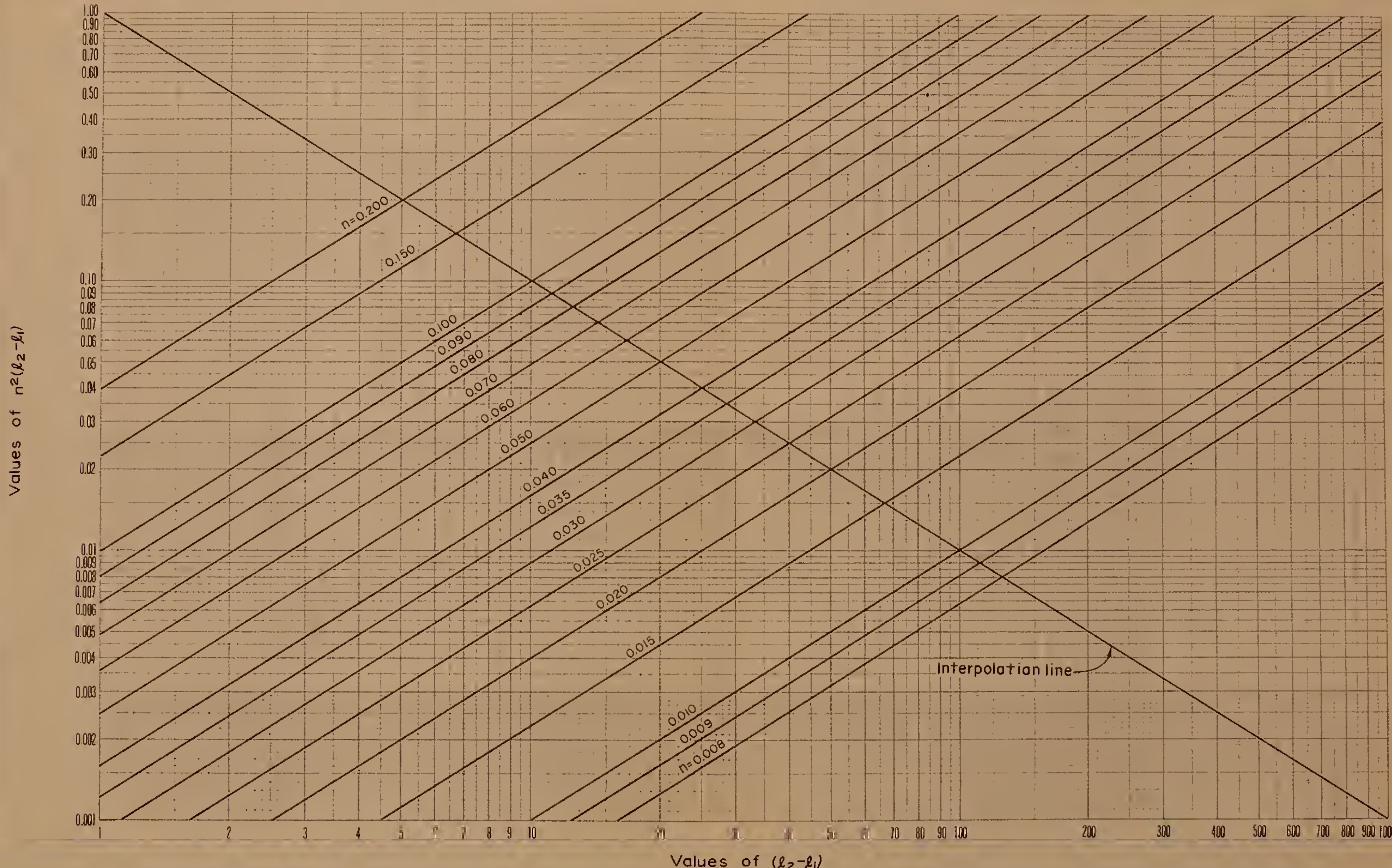
a. When the <u>discharge Q is given</u> and the <u>starting depth is not given</u> (that is a tailwater or headwater depth) and no structures are involved at the starting end of the channel under consideration, the starting depth may be determined by the procedure given on sheets 4, 5, and 6 of ES-142. In this situation it is necessary to know the tailwater condition when flow is subcritical and the headwater condition when flow is supercritical to determine the starting depth. This requires that sufficient length of channel beyond the channel under consideration be given. When the downstream end section of the channel under consideration having subcritical flow terminates at a structure or pool, the starting depth can be determined by the hydraulic characteristics of the structure or the elevation of the pool.

b. When the <u>discharge Q is not given</u> and the <u>starting depth is given</u>, the discharge Q is determinable in subcritical flow if the headwater depth (or some other elevation condition) is given.

If the starting depth is given in supercritical flow, the tailwater depth (or some other elevation condition) must also be given to determine the discharge Q. Select various discharges, and compute their corresponding water surface profiles from the given starting depth. A rating curve is prepared at the station of the given elevation condition to determine the discharge Q. Sometimes the discharge is controlled by a structure or by a control section at the upstream end of the channel under consideration for the given "elevation condition", and the tailwater has no effect on the discharge. In these situations the discharge is determined without recourse to water surface profile determination.

c. When the discharge Q and the starting depth are not given, an elevation condition other than what would normally be used for a starting depth must be given. The discharge Q and starting depth can be determined for only those cases in which no hydraulic jump occurs between the starting depth to be determined and the known elevation condition. Various discharges are selected, and their starting depths are determined by the procedure given on sheets 4, 5, and 6 of ES-142. The corresponding water surface profiles for the selected discharges and their corresponding starting depths are computed. A rating curve is prepared at the station of the given elevation condition to determine the discharge Q.

HYDRAULICS: $n^2(\ell_2-\ell_1)$ vs. $(\ell_2-\ell_1)$ for various values of n

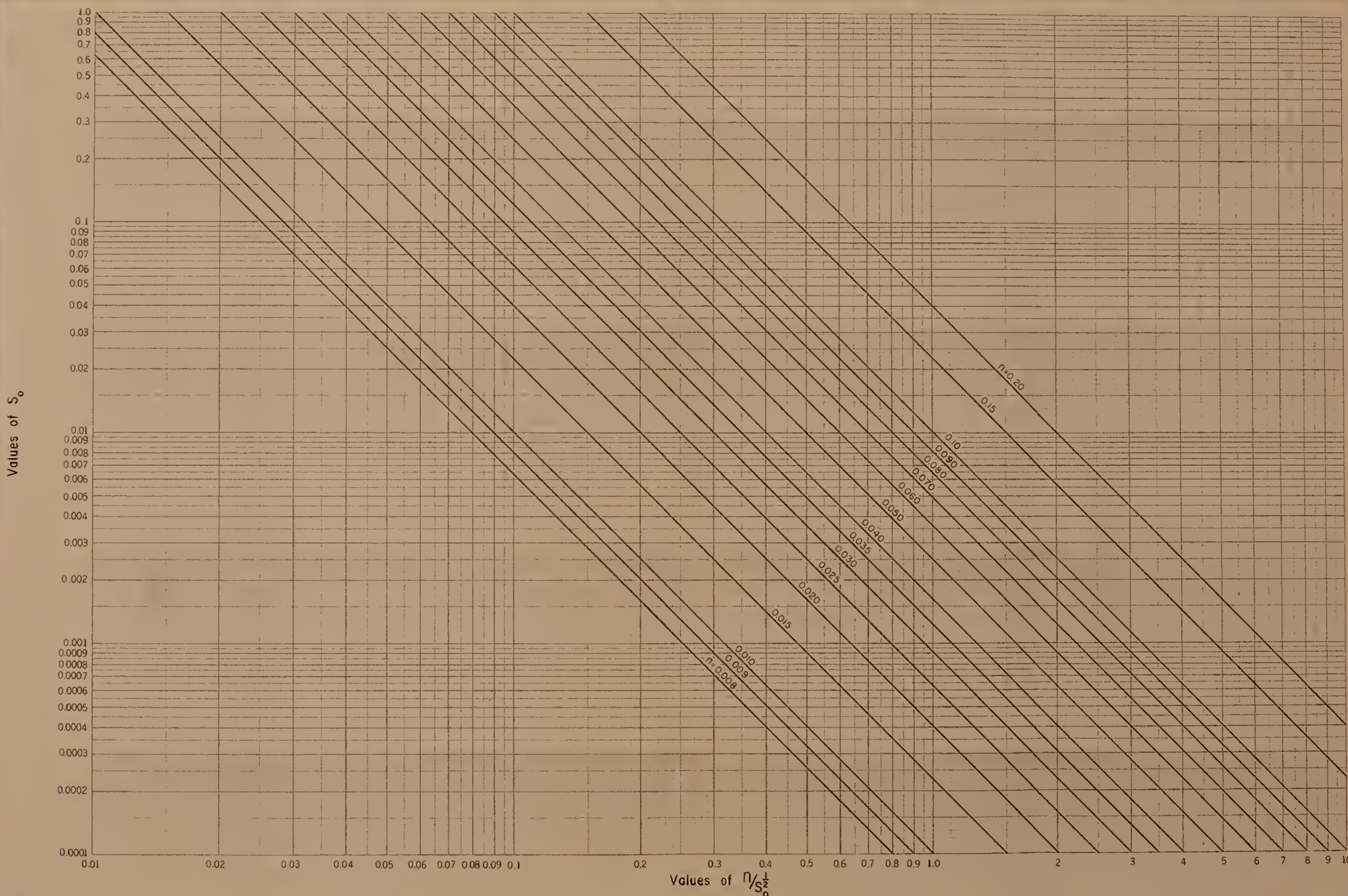


REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO.
ES- 137
SHEET 1 OF 1
DATE 3-1-60

HYDRAULICS: S_o vs. $n/S_o^{1/2}$ for various values of n

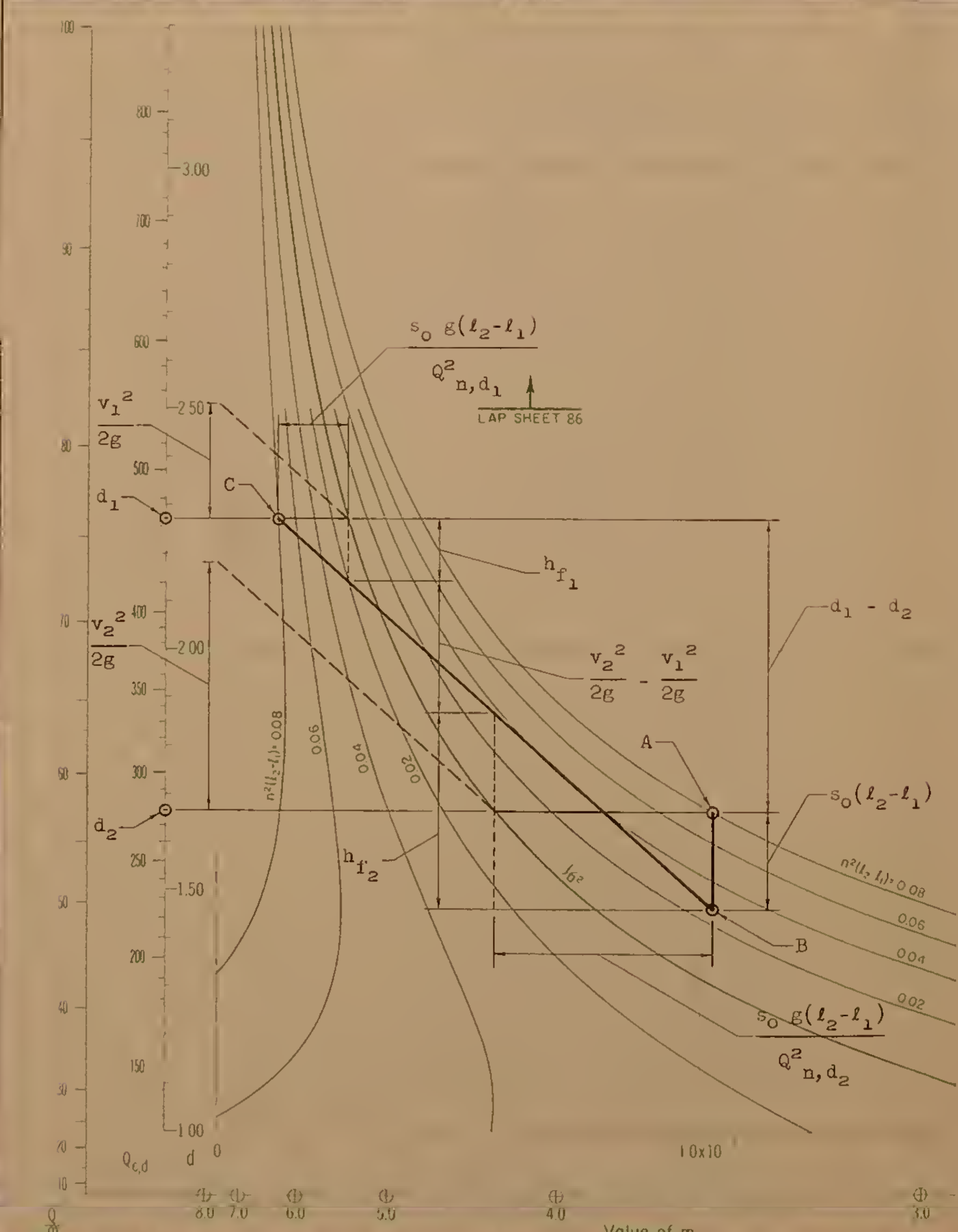


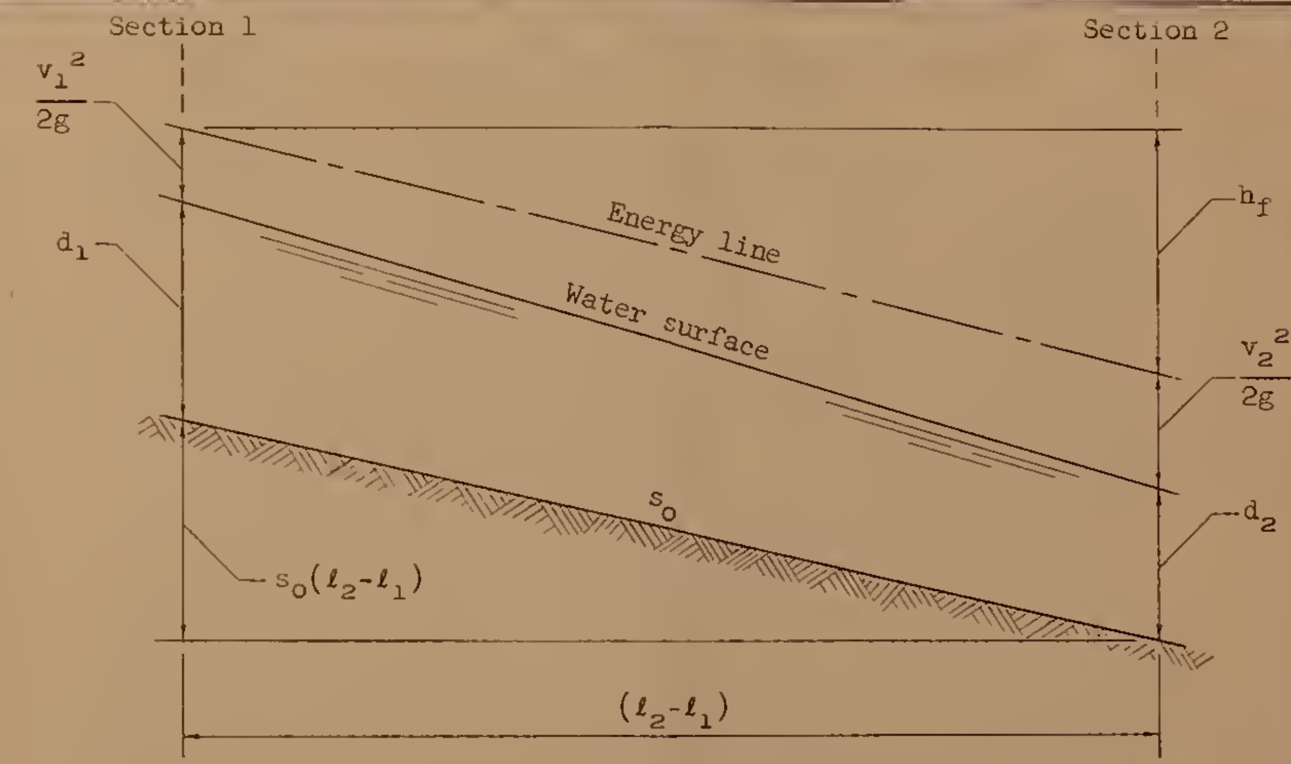
REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO.
ES- 138
SHEET 1 OF 1
DATE 3-1-60

HYDRAULICS: Energy relations of varied flow





$$d_1 + \frac{v_1^2}{2g} + s_o(\ell_2-\ell_1) = d_2 + \frac{v_2^2}{2g} + h_f$$

or as shown on the left drawing

$$\frac{v_2^2}{2g} - \frac{v_1^2}{2g} + h_f = d_1 - d_2 + s_o(\ell_2-\ell_1)$$

$$h_f = h_{f_1} + h_{f_2}$$

$$h_{f_1} = \frac{Q^2}{2g}\left[\frac{s_o\ g(\ell_2-\ell_1)}{Q^2_{n,d_1}}\right]$$

$$h_{f_2} = \frac{Q^2}{2g}\left[\frac{s_o\ g(\ell_2-\ell_1)}{Q^2_{n,d_2}}\right]$$

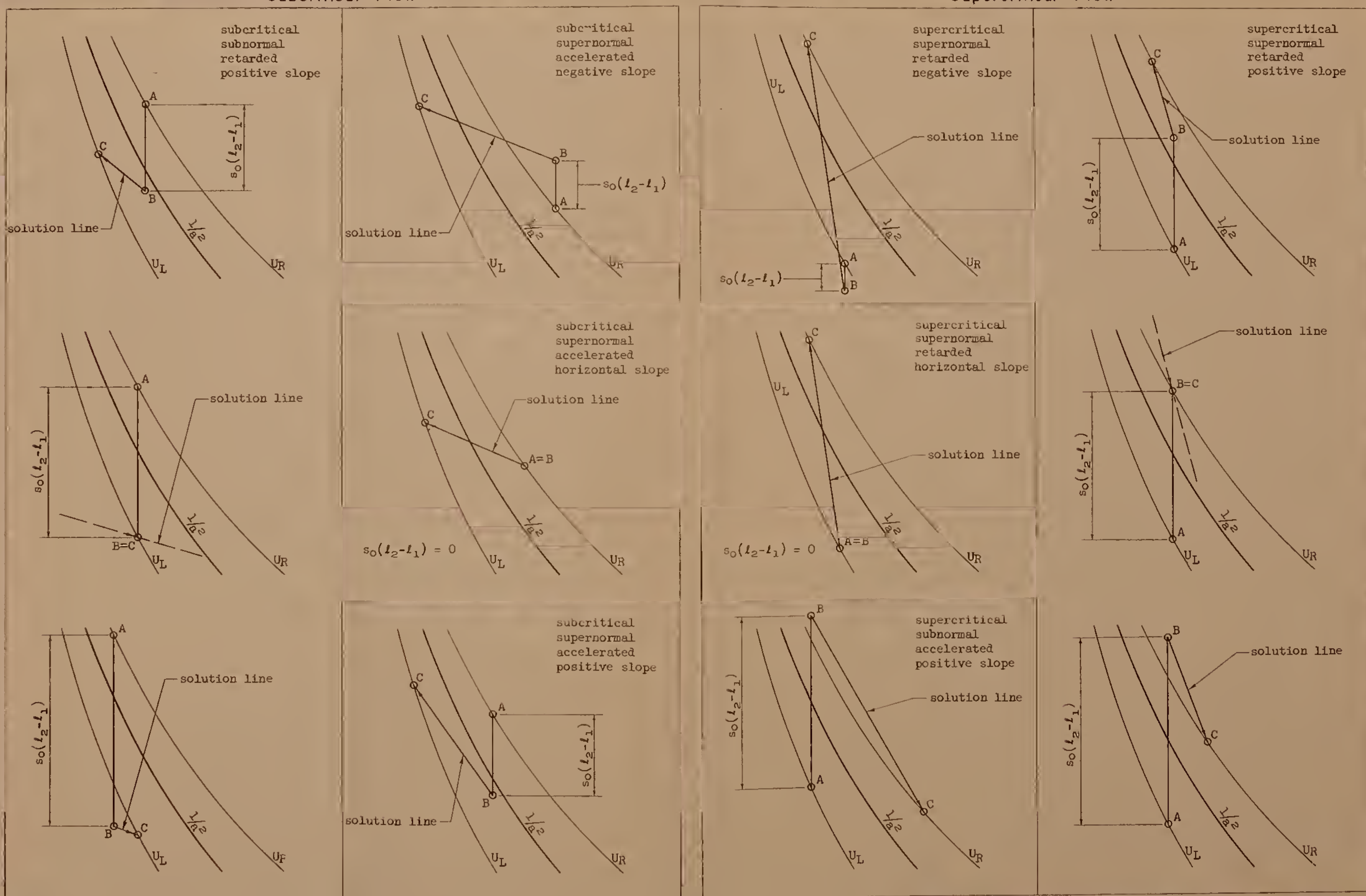
REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION · DESIGN SECTION

STANDARD DWG. NO.
ES- 142
SHEET 1 OF 6
DATE 8-10-60

HYDRAULICS: Summary of graphic procedures on Water Surface Profile charts (ES-125 thru ES-134) for prismatic channels

Subcritical Flow — Supercritical Flow



REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING DIVISION - DESIGN SECTION	STANDARD DWG. NO. ES- 142 SHEET 2 OF 6 DATE 8-10-60

HYDRAULICS: Procedure for computing the unknown depth at the end of a computational reach, when the starting depth is known.

REFERENCE

Begin

Select the proper sheet of ES-4 charts (ES-125 through ES-155) according to the given values of z, b and starting depth d.

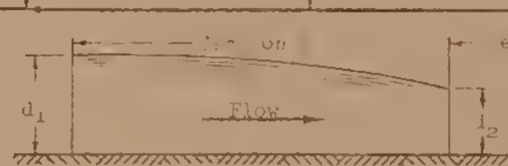
Select a value of $n^2(l_2-l_1)$ from chart.

Determine the computational reach (l_2-l_1) from ES-157 using the given value of Manning's coefficient n and the selected value of $n^2(l_2-l_1)$. The computational reach (l_2-l_1) must not include a break in grade.

Compute $s_o(l_2-l_1)$ using (l_2-l_1) as determined in the preceding step and the given bottom slope s_o.

Determine $d_{c,Q}$.

$d < d_{c,Q}$	$d = d_{c,Q}$	$d > d_{c,Q}$



$d < d_{c,Q}$:

Water surface profile is to be computed downstream.

Determine the point A. Point A is the intersection of the given depth $d = d_1$ and U_L curve having the selected value of $n^2(l_2-l_1)$ used in determining $s_o(l_2-l_1)$.

Scale the distance $s_o(l_2-l_1)$ vertically from point A to locate point B. For positive values of s_o scale upward from point A, and for negative values of s_o scale downward from point A. If s_o is zero, point A is coincident with point B. (See sheet 2.)

Determine the direction of the solution line and draw it from point B. Does it intersect the U_R curve which is conjugate to the U_L curve used to determine point A?

No	Yes

Does the solution line drawn through point B intersect the U_R curve at a depth equal to or less than $d_{c,Q}$?

No	Yes

Designate this depth as d_2 and the intersection at this depth as point C.

Consider the horizontal line segment extending from the point A to the $\frac{1}{a^2}$ curve. Project it vertically to the solution line. The vertical component of this projected line segment is the friction head loss h_{f_1} attributed to the depth of flow d_1 for the distance $\frac{l_2-l_1}{2}$. Consider the horizontal line segment extending from the point C to the $\frac{1}{a^2}$ curve. Project it vertically to the solution line. The vertical component of this projected line segment is the friction head loss h_{f_2} attributed to the depth of flow d_2 for the distance $\frac{l_2-l_1}{2}$. (See sheet 1.)

$d > d_{c,Q}$:

Water surface profile is to be computed upstream.

Determine the point A. Point A is the intersection of the given depth $d = d_2$ and the U_R curve having the selected value of $n^2(l_2-l_1)$ used in determining $s_o(l_2-l_1)$.

Scale the distance $s_o(l_2-l_1)$ vertically from point A to locate point B. For positive values of s_o scale downward from point A, and for negative values of s_o scale upward from point A. If s_o is zero, point A is coincident with point B. (See sheet 2.)

Determine the direction of the solution line and draw it from point B. Does it intersect the U_L curve which is conjugate to the U_R curve used to determine point A?

No	Yes

Does the solution line drawn through point B intersect the U_L curve at a depth equal to or greater than $d_{c,Q}$?

No	Yes

Designate this depth as d_1 and the intersection at this depth as point C.

Consider the horizontal line segment extending from the point A to the $\frac{1}{a^2}$ curve. Project it vertically to the solution line. The vertical component of this projected line segment is the friction head loss h_{f_2} attributed to the depth of flow d_2 for the distance $\frac{l_2-l_1}{2}$. Consider the horizontal line segment extending from the point C to the $\frac{1}{a^2}$ curve. Project it vertically to the solution line. The vertical component of this projected line segment is the friction head loss h_{f_1} attributed to the depth of flow d_1 for the distance $\frac{l_2-l_1}{2}$. (See sheet 1.)

Both branches:

Compute the value of $|h_{f_2} - h_{f_1}|$. (See sheet 1.)

Is the difference in the values of h_{f_2} and h_{f_1} greater than the degree of accuracy desired?

Yes	No

Yes: Select a shorter computational reach by selecting a smaller value of $n^2(l_2-l_1)$.

No: Is another computational reach required?

No	Yes

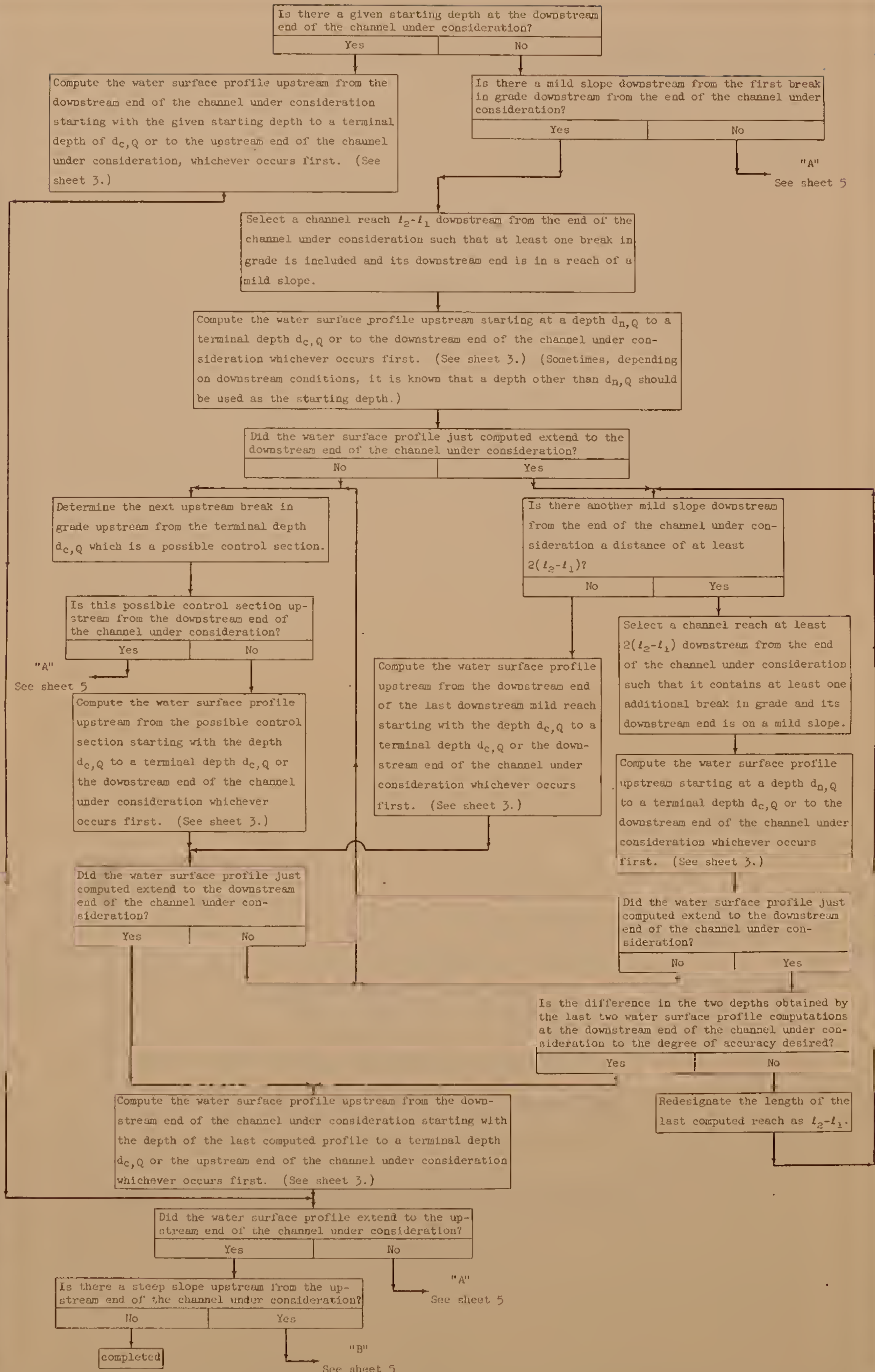
No: Follow the next procedure given by sheets 4, 5 or 6.

Yes: Redesignate the last computed depth d_1 or d_2 as the new starting depth "d".

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO.
ES- 142
SHEET 3 OF 6
DATE 8-10-60

HYDRAULICS: Procedure for computing a water surface profile for a channel under consideration when the starting depth is or is not given.

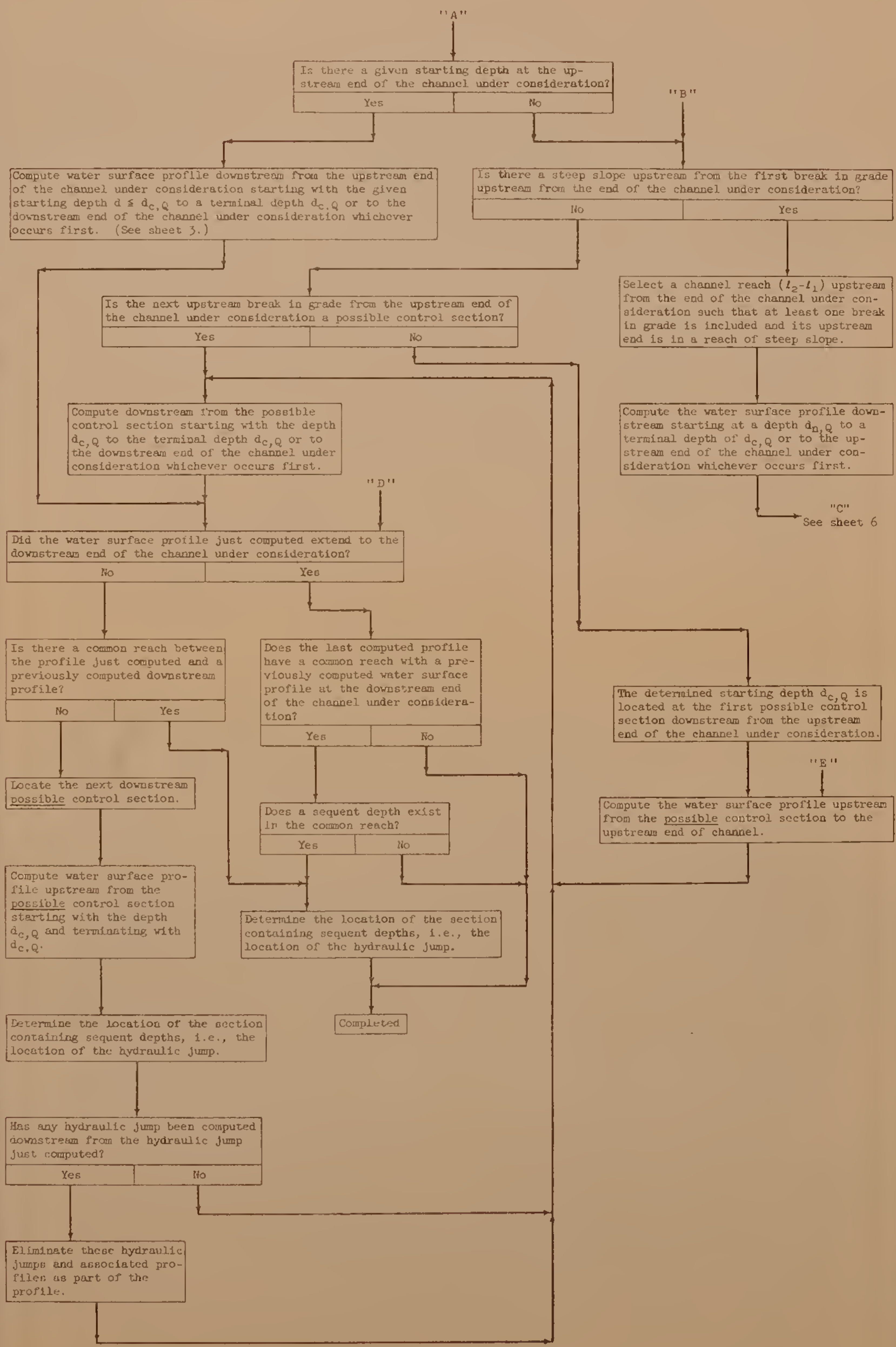


REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO.
ES-142
SHEET 4 OF 6
DATE 8-10-60

HYDRAULICS: Procedure for computing a water surface profile for a channel under consideration when the starting depth is or is not given.



REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
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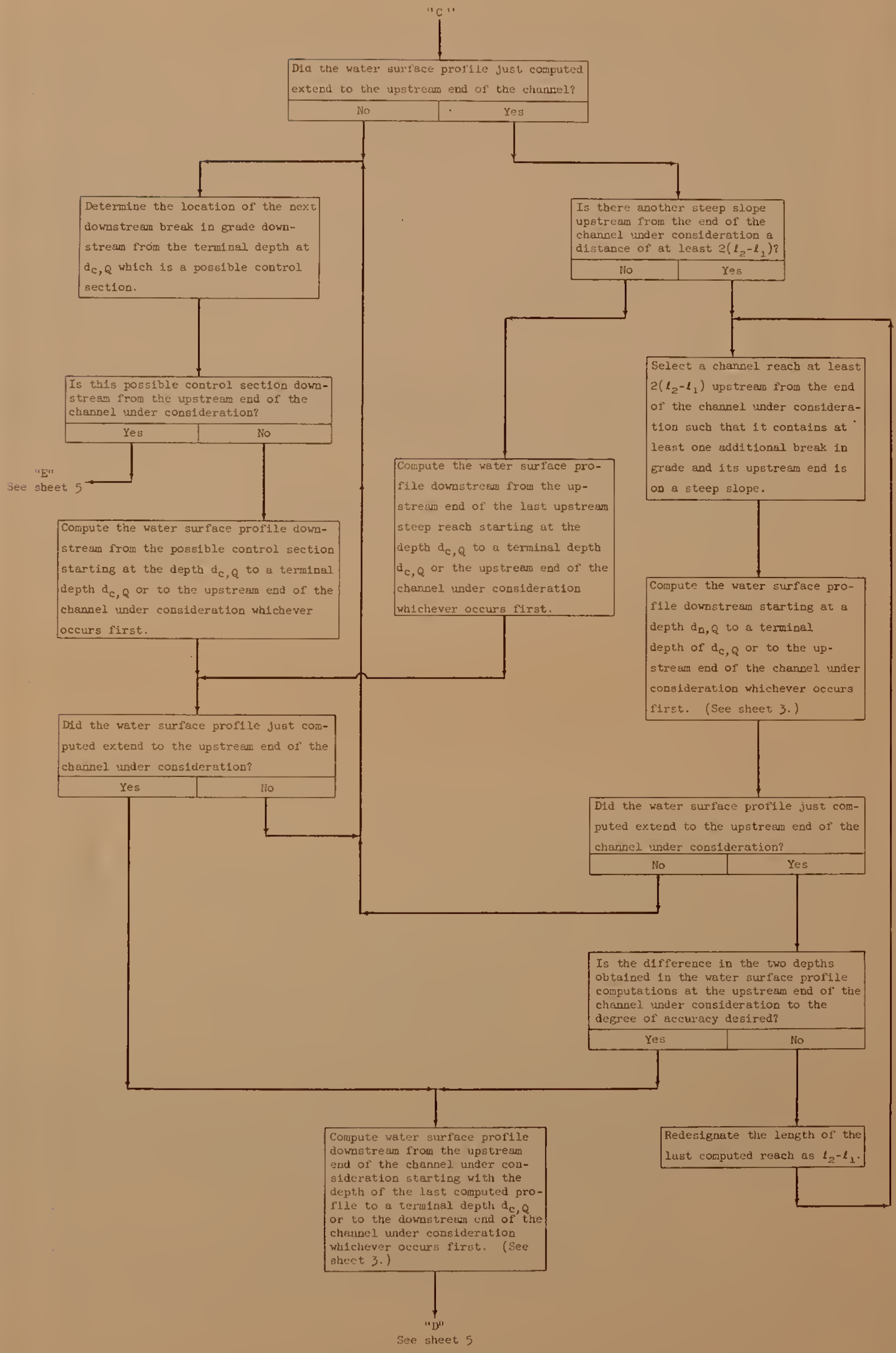
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO.
ES-142
SHEET 5 OF 6
DATE 8-10-60

HYDRAULICS: Procedure for computing a water surface profile for a channel under consideration when the starting depth is or is not given.



REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO.
ES-142
SHEET 6 OF 6
DATE 8-10-60

CHAPTER 4 - EXAMPLES

EXAMPLE 1

Given: (1) Trapezoidal channel, $z = 2$
(2) $b = 50$ ft
(3) $d = 3.2$ ft

Determine: The critical discharge $Q_{c,d}$ corresponding to the given depth.

Solution: Select the proper chart (ES-129, sheet 140). On the left-hand line chart ($Q_{c,d}$ vs d) for a value of $d = 3.2$ ft, read $Q_{c,d} = 1740$ cfs.

EXAMPLE 2

Given: (1) Trapezoidal channel, $z = 2$
(2) $b = 50$ ft
(3) $Q = 1700$ cfs

Determine: The critical depth $d_{c,Q}$ corresponding to the given discharge.

Solution: Select the proper chart (ES-129, sheet 140). Mentally redesignate the parameters ($Q_{c,d}$ vs d) of the left-hand line chart as (Q vs $d_{c,Q}$). For $Q = 1700$ cfs, read $d_{c,Q} = 3.16$ ft.

EXAMPLE 3

Given: (1) Trapezoidal channel, $z = 2$
(2) $b = 50$ ft
(3) $d = 3.9$ ft
(4) $s_o = 0.003$ ft/ft
(5) $n = 0.035$

Determine: The normal discharge $Q_{n,d}$ corresponding to the given depth.

Solution: Select the proper chart (ES-129, sheet 140). On the right-hand line chart $\left(d \text{ vs } \left[\frac{nQ_{n,d}}{s_o^{1/2}}\right]\right)$ for $d = 3.9$ ft, read $\left[\frac{nQ_{n,d}}{s_o^{1/2}}\right] = 750$.

Obtain $\left[\frac{n}{s_o^{1/2}}\right] = 0.64$ from ES-138.

$$\text{Solve for } Q_{n,d} = \frac{\left[\frac{nQ_{n,d}}{s_o^{1/2}}\right]}{\left[\frac{n}{s_o^{1/2}}\right]} = \frac{750}{0.64} = 1172 \text{ cfs.}$$

EXAMPLE 4

Given: (1) Trapezoidal channel, $z = 2$
(2) $b = 50$ ft
(3) $s_o = 0.003$ ft/ft
(4) $Q = 1500$ cfs
(5) $n = 0.035$

Determine: The normal depth $d_{n,Q}$ corresponding to the given discharge.

Solution: From ES-138 find $\left[\frac{n}{s_o^{1/2}}\right] = 0.64$

Compute $\left[\frac{nQ}{s_o^{1/2}}\right] = \left[\frac{n}{s_o^{1/2}}\right] Q = (0.64)(1500) = 960.$

Select the proper chart (ES-129, sheet 140).

Mentally redesignate the parameters $\left(d \text{ vs } \left[\frac{n\ Q_{n,d}}{s_o^{1/2}}\right]\right)$ of the right-hand line chart as $\left(d_{n,Q} \text{ vs } \left[\frac{nQ}{s_o^{1/2}}\right]\right)$. For $\left[\frac{nQ}{s_o^{1/2}}\right] = 960$, read $d_{n,Q} = 4.50$ ft.

EXAMPLE 5

Given: (1) Trapezoidal channel, $z = 2$
(2) $b = 50$ ft
(3) $n = 0.04$
(4) $d = 4.16$ ft
(5) $Q = 1700$ cfs

Determine: The normal slope $s_{n,Qd}$ corresponding to the given discharge and depth.

Solution: Locate the proper chart (ES-129, sheet 140).

Mentally redesignate the parameters $\left(d \text{ vs } \left[\frac{n\ Q_{n,d}}{s_o^{1/2}}\right]\right)$ of the right-hand line chart as $\left(d \text{ vs } \left[\frac{nQ}{(s_{n,Qd})^{1/2}}\right]\right)$. For $d = 4.16$ ft, read

$$\left[\frac{nQ}{(s_{n,Qd})^{1/2}}\right] = 835.$$

(Method 2.) Compute $\left[\frac{n}{(s_{n,Qd})^{1/2}}\right] = \left[\frac{nQ}{(s_{n,Qd})^{1/2}}\right] \frac{1}{Q} = \frac{835}{1700} = 0.491$

Mentally redesignate the parameters $\left(s_o,\ n,\ \text{and}\ \left[\frac{n}{s_o^{1/2}}\right]\right)$ of ES-138 as $\left(s_{n,Qd},\ n,\ \text{and}\left[\frac{n}{(s_{n,Qd})^{1/2}}\right]\right)$. For $\left[\frac{n}{(s_{n,Qd})^{1/2}}\right] = 0.491$ and $n = 0.04$, read $s_{n,Qd} = 0.0066$ ft/ft.

EXAMPLE 6

Given: (1) Trapezoidal channel, $z = 2$
(2) $b = 50$ ft
(3) $n = 0.017$
(4) $d = 4.53$ ft

Determine: The critical slope $s_{c,d}$ corresponding to the given depth.

Solution: Select the proper chart. (ES-129, sheet 140).
On the left-hand line chart ($Q_{c,d}$ vs d) for $d = 4.53$ ft, read $Q_{c,d} = 3000$ cfs.

Mentally redesignate the parameters $\left(d\ \text{vs}\ \left[\frac{n\ Q_{n,d}}{s_o^{1/2}}\right]\right)$ of the right-hand line chart as $\left(d\ \text{vs}\ \left[\frac{n\ Q_{c,d}}{(s_{c,d})^{1/2}}\right]\right)$.

For $d = 4.53$ ft, read $\left[\frac{n\ Q_{c,d}}{(s_{c,d})^{1/2}}\right] = 970$.

(Method 2.) Compute $\left[\frac{n}{(s_{c,d})^{1/2}}\right] = \left[\frac{n\ Q_{c,d}}{(s_{c,d})^{1/2}}\right] \frac{1}{Q_{c,d}} = \frac{970}{3000} = 0.323$

Mentally redesignate the parameters $\left(s_o,\ n,\ \text{and}\left[\frac{n}{s_o^{1/2}}\right]\right)$ of ES-138 as $\left(s_{c,d},\ n,\ \text{and}\left[\frac{n}{(s_{c,d})^{1/2}}\right]\right)$.

For $\left[\frac{n}{(s_{c,d})^{1/2}}\right] = 0.325$ and $n = 0.017$, read $s_{c,d} = 0.00277$ ft/ft.

EXAMPLE 7

Given: (1) Trapezoidal channel, $z = 2$
(2) $b = 50$ ft
(3) $n = 0.017$
(4) $Q = 3000$ cfs

Determine: The critical slope $s_{c,Q}$ corresponding to the given discharge.

Solution: Select the proper chart. (ES-129, sheet 140). Mentally redesignate the parameters ($Q_{c,d}$ vs d) of the left-hand line chart as (Q vs $d_{c,Q}$). For Q = 3000 cfs, read $d_{c,Q}$ = 4.53 ft.

Mentally redesignate the parameters $\left(d \text{ vs } \left[\frac{n\,Q_{n,d}}{s_o^{1/2}}\right]\right)$ of the right-hand line chart as $\left(d_{c,Q} \text{ vs } \left[\frac{nQ}{(s_{c,Q})^{1/2}}\right]\right)$. For $d_{c,Q}$ = 4.53 ft, read

$$\left[\frac{nQ}{(s_{c,Q})^{1/2}}\right] = 970.$$

(Method 2.) Compute $\left[\frac{n}{(s_{c,Q})^{1/2}}\right] = \left[\frac{nQ}{(s_{c,Q})^{1/2}}\right]\frac{1}{Q} = \frac{970}{3000} = 0.323$

Mentally redesignate the parameters $\left(s_o,\ n, \text{ and } \left[\frac{n}{s_o^{1/2}}\right]\right)$ of ES-138 as $\left(s_{c,Q},\ n, \text{ and } \left[\frac{n}{(s_{c,Q})^{1/2}}\right]\right)$. For $\left[\frac{n}{(s_{c,Q})^{1/2}}\right]$ = 0.323 and n = 0.017, read $s_{c,Q}$ = 0.00277 ft/ft.

EXAMPLE 8

Given: (1) Trapezoidal channel, z = 2
(2) b = 50 ft
(3) n = 0.020
(4) d = 4.50 ft
(5) Q = 2000 cfs

Determine: The rate of friction loss $\frac{dh_f}{d\ell}$ corresponding to the given depth and discharge.

Solution: Locate the proper sheet (ES-129, sheet 140). On the right-hand line chart $\left(\left[\frac{nQ_{n,d}}{s_o^{1/2}}\right] \text{ vs } d\right)$ for d = 4.5 ft, read $\left[\frac{nQ_{n,d}}{s_o^{1/2}}\right]$ = 957.

Compute $\left[\frac{Q_{n,d}^2}{s_o}\right] = \left(\left[\frac{nQ_{n,d}}{s_o^{1/2}}\right]\frac{1}{n}\right)^2 = \left(\frac{957}{0.020}\right)^2 = 2{,}290{,}000{,}000.$

Compute $s = s_o\left[\frac{Q}{Q_{n,d}}\right]^2 = \frac{Q^2}{\left[\frac{Q_{n,d}^2}{s_o}\right]} = \frac{4{,}000{,}000}{2{,}290{,}000{,}000} = 0.001747$ ft/ft.

EXAMPLE 9

Given: (1) Trapezoidal channel, $z = 2$
(2) $b = 50$ ft
(3) $Q = 2100$ cfs

Determine: The direction of the solution line for the given discharge.

Solution: Select the proper sheet (ES-129, sheet 140). The smallest value of m is 3.0 Compute $\frac{Q}{m} = \frac{2100}{3} = 700$ which can be read on the $\frac{Q}{m}$ scale. The direction of the solution line corresponding to the discharge 2100 cfs is parallel to the vector drawn from the point $m = 3$ to the point $\frac{Q}{m} = 700$.

EXAMPLE 10

Given: (1) Trapezoidal channel, $z = 2$
(2) $b = 50$ ft
(3) Direction of the solution line defined by the two points:
a. $\frac{1}{a^2}$ curve at $d = 4$ ft and
b. axis of ordinates at the value of 7 ft.

Determine: The discharge Q corresponding to the given direction of the solution line.

Solution: Select the proper chart (ES-129, sheet 140). Locate the two given points which define the given direction of the solution line.

Draw a line showing this direction from $m = 4$ to intersect the $\frac{Q}{m}$ scale at the value $\frac{Q}{m} = 805$. Multiply the value $\frac{Q}{m} = 805$ by $m = 4.0$ to obtain the corresponding discharge $Q = 3220$ cfs.

EXAMPLE 11

Given: (1) Trapezoidal channel, $z = 2$
(2) $b = 50$ ft
(3) $Q = 2100$ cfs
(4) $d = 3.8$ ft

Determine: The specific energy head $H_{e,Qd}$ corresponding to the given depth and discharge.

Solution: Select the proper chart (ES-129, sheet 140). Determine the direction of the solution line for the discharge $Q = 2100$ cfs. (See example 9.) Construct a line having the direction of the solution line from the $\frac{1}{a^2}$ curve at $d = 3.8$ to intersect the axis of ordinates. The value read at this point of intersection is the specific energy head, $H_{e,Qd} = 5.24$ feet.

EXAMPLE 12

Given: (1) Trapezoidal channel, $z = 2$
(2) $b = 50$ ft
(3) $H_{e,Q} = 5.6$ ft
(4) $Q = 2100$ cfs

Determine: The depths d at which the given discharge may occur in the given channel at the given specific energy head $H_{e,Q}$.

Solution: Select the proper chart (ES-129, sheet 140). Determine the direction of the solution line for the given discharge. (See example 9.) Draw a line having the direction of the solution line from the value $H_{e,Q} = 5.6$ ft on the axis of ordinates to intersect the $\frac{1}{a^2}$ curve at two points. The depths read at the points of intersection are:

$d = 2.83$ ft and
$d = 4.74$ ft.

EXAMPLE 13

Given: (1) Trapezoidal channel, $z = 2$
(2) $b = 50$ ft
(3) $d = 4.74$ ft
(4) $H_{e,d} = 5.6$ ft

Determine: The discharge Q corresponding to the given depth and the given specific energy head.

Solution: Select the proper chart (ES-129, sheet 140). Draw the line from the value of 5.6 ft on the axis of ordinates to the $\frac{1}{a^2}$ curve at the depth 4.74 ft. Determine the discharge Q corresponding to the direction of this line. (See procedure 10.) On selecting $m = 3$, the value of $\frac{Q}{m}$ is equal to 700. Hence, the corresponding discharge $Q = 700 \times 3 = 2100$ cfs.

EXAMPLE 14

Given: (1) Trapezoidal channel, $z = 2$
(2) $b = 50$ ft
(3) $H_e = 5.5$ ft

Determine: The critical discharge Q_c and the critical depth d_c corresponding to the specific energy head $H_e = 5.50$ ft.

Solution: Select the proper chart (ES-129, sheet 140). Mentally redesignate the axis of ordinates of the central chart d as H_e. From the point $H_e = 5.50$ ft on the axis of ordinates, draw the line tangent to the $\frac{1}{a^2}$ curve. Determine Q_c for this given direction of the solution line. $Q_c = 2286$ cfs. (See procedure 10.)

Mentally redesignate the parameters (d vs. $Q_{c,d}$) of the left-hand line chart as (d_c vs. Q_c). For $Q_c = 2286$ cfs, read $d_c = 3.82$ ft. (See procedure 2.)

EXAMPLE 15

Given: (1) Trapezoidal channel, $z = 2$
(2) $b = 50$ ft
(3) $Q = 2100$ cfs
(4) $d = 3.8$ ft

Determine: The velocity head $\left[\frac{v^2}{2g}\right]$ of the given channel corresponding to the given depth and discharge.

Solution: Select the proper chart (ES-129, sheet 140). Determine the specific energy head $H_{e,Qd}$ corresponding to the given depth and discharge. (See example 11.)

$$H_{e,Qd} \equiv d + \left[\frac{v^2}{2g}\right] = 5.24 \text{ ft}$$

$$\left[\frac{v^2}{2g}\right] = H_{e,Qd} - d = 5.24 - 3.80 = 1.44 \text{ ft}$$

EXAMPLE 16

Given: (1) Trapezoidal channel, $z = 2$
(2) $b = 50$ ft
(3) $n = 0.020$
(4) $s_o = 0.0025$ ft/ft

Determine: The depth d and discharge Q corresponding to the critical slope equal to s_o.

Solution: Select the proper charts (ES-129, sheets 139 through 144). Determine $Q_{c,d}$ and $Q_{n,d}$ for several values of d, using a tabular form. Column 2 is read directly from the left-hand line chart, and Column 3 is read directly from the right-hand line chart.

Compute $\left[\frac{n}{s_o^{1/2}}\right] = \left[\frac{0.020}{(0.0025)^{1/2}}\right] = 0.40$

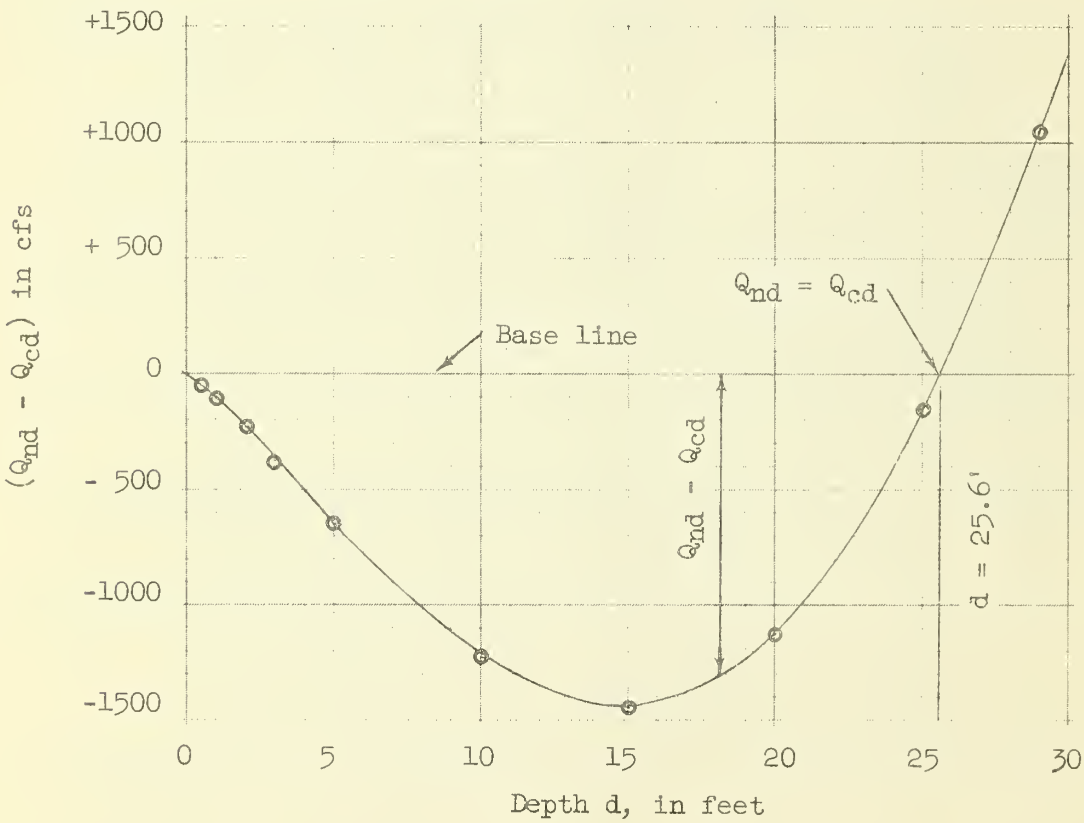
Column 4 is Column 3 divided by the computed value of $\left[\frac{n}{s_o^{1/2}}\right]$.

Column 5 is the difference of $Q_{c,d}$ and $Q_{n,d}$ for each depth d.

Since these differences are small in relation to the total discharges, the differences are plotted against depth to find a more accurate intersection of the points where $d_{c,Q} = d_{n,Q}$.

(1)	(2)	(3)	(4)	(5)
d	$Q_{c,d}$	$\left[\frac{n\, Q_{n,d}}{s_o^{1/2}}\right]$	$Q_{n,d}$	$(Q_{n,d} - Q_{c,d})$
29	77,200	31,300	78,250	+ 1050
25	57,900	23,100	57,750	- 150
20	38,000	14,750	36,875	- 1125
15	22,500	8,420	21,050	- 1450
10	11,100	3,950	9,875	- 1225
5	3,525	1,150	2,875	- 650
3	1,570	476	1,190	- 380
2	837	242	605	- 232
1	289	75	187	- 102
.5	101	23.5	59	- 42
.2	25.8	5	12.5	- 13.3

Plot the curve $\left((Q_{n,d} - Q_{c,d}) \text{ vs. } d\right)$.



Read the depth $d = 25.6$ ft corresponding to the critical slope equal to s_o from the plotted curve at $(Q_{n,d} - Q_{c,d}) = 0$. Read the discharge Q corresponding to the depth of 25.6 ft on the given slope directly from the left-hand line chart.

The discharge Q, for which the given bottom slope s_o is critical, is $Q = 60,600$ cfs.

EXAMPLE 17

Given:
(1) Trapezoidal channel, $z = 2$
(2) $b = 50$ ft
(3) $s_o = 0.005$ ft/ft
(4) $Q = 1825$ cfs
(5) $n = 0.040$
(6) $d = 4.15$ ft
(7) Tolerable error of 0.05 ft

Determine: The unknown depth at the distance $(\ell_2-\ell_1)$ from the section of known depth.

Solution: Select the proper chart (ES-129, sheet 140).

Select $n^2(\ell_2-\ell_1) = 0.10$ From ES-137, $(\ell_2-\ell_1) = 62.5$ ft.

Compute $s_o(\ell_2-\ell_1) = 0.005\ (62.5) = 0.312$ ft.

Read $d_{c,Q}$ corresponding to the discharge $Q = 1825$ cfs on the left-hand line chart.

$$d_{c,Q} = 3.30 \text{ ft} < d = 4.15 \text{ ft}$$

Thus, flow is subcritical and water surface profile computations are made in an upstream direction.

Find point A on the chart at $d = 4.15$ ft on the U_R-curve for $n^2(\ell_2-\ell_1) = 0.10$ (See page 4-10.)

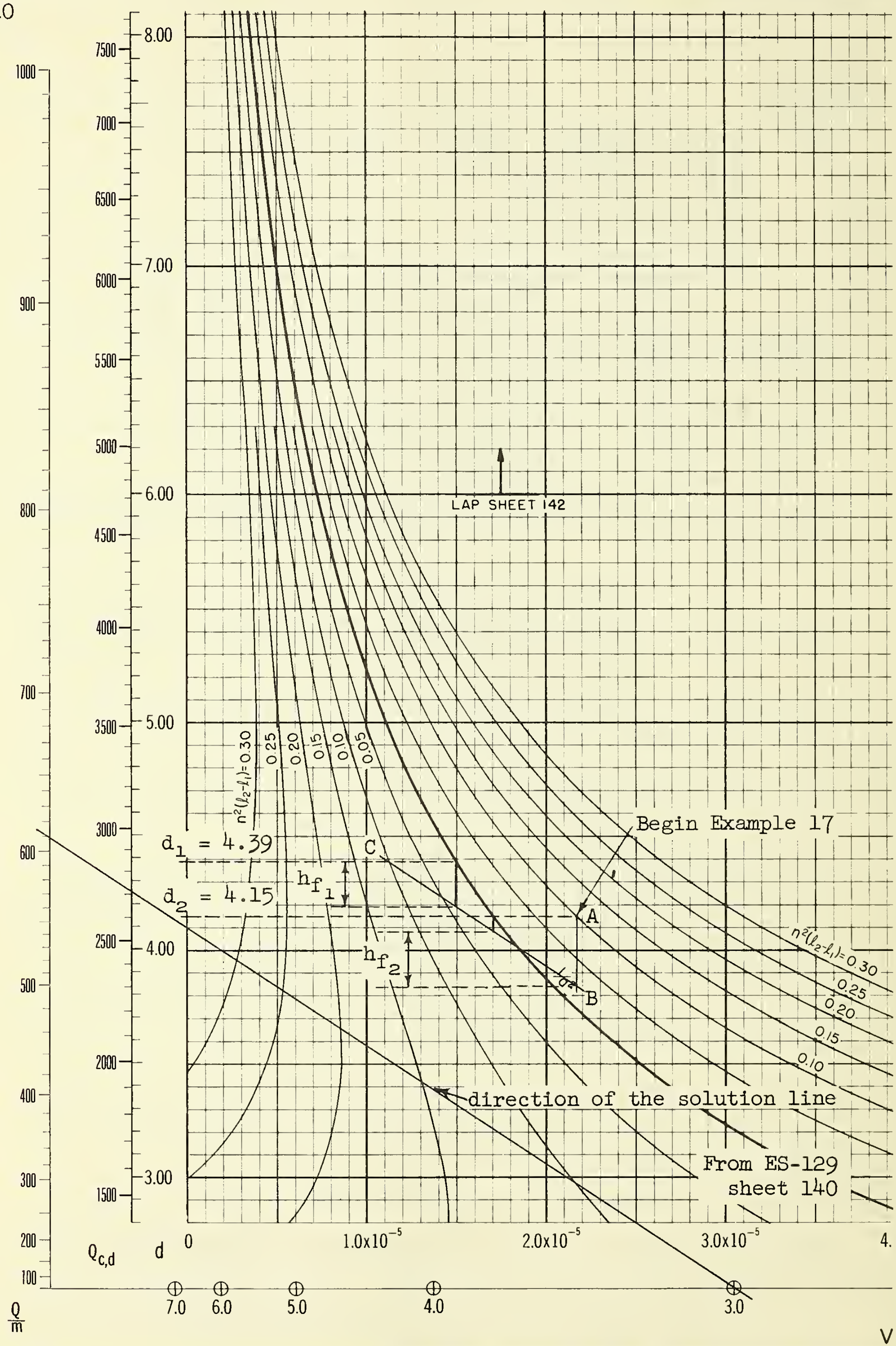
Scale the distance $s_o(\ell_2-\ell_1) = 0.312$ down from point A. This is point B, at $d = 3.838$ ft.

Determine the direction of the solution line through points $m = 3$ on the m-scale and $\frac{Q}{m} = 608$ on the $\frac{Q}{m}$ scale. Draw the solution line through point B to intersect the conjugate U_L-curve $\left(n^2(\ell_2-\ell_1) = 0.10\right)$. Read the depth d_1, which is greater than $d_{c,Q}$, as 4.39 ft. This is the depth at the station 62.5 ft upstream from the section at which d_2 is 4.15 ft.

Project the horizontal line segment between the $\frac{1}{a^2}$ curve and A to the solution line. The vertical component of this projected line segment is $4.08 - 3.84 = 0.24 = h_{f_2}$.

Project the horizontal line segment between C and the $\frac{1}{a^2}$ curve to the solution line. The vertical component of this projected line segment is $4.39 - 4.19 = 0.20 = h_{f_1}$.

This solution has an error of less than $|h_{f_2} - h_{f_1}| = |0.24 - 0.20| = 0.40$ ft, which is less than the tolerable error of 0.05 ft.

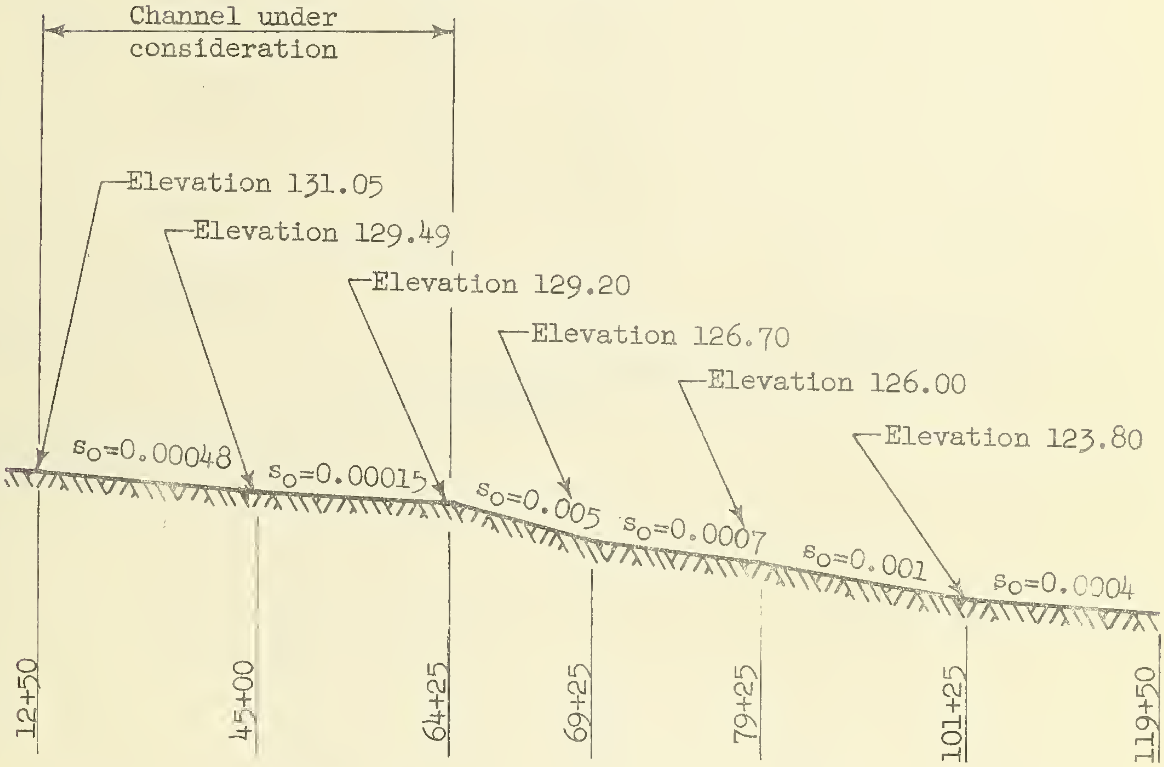


REFERENCE

These Water Surface Profile Charts were developed and computed by Paul D. Daubt, head, Design Section.

EXAMPLE 18a

Given: (1) Trapezoidal channel, $z = 2$
(2) $b = 45$ ft
(3) The profile of the channel bottom as shown in the figure
(4) $Q = 4000$ cfs
(5) $n = 0.020$
(6) The elevation of a lake, into which the channel discharges, is 119.4 ft at approximately six miles downstream from the channel under consideration. The channel slopes were observed to be generally uniform all the way to the lake.
(7) Tolerable error of 0.05 ft.



Determine: The water surface profile for the channel under consideration.

Solution: Since there is no given starting depth, the procedure given on sheet 4 of ES-142 is used in the solution of this example.

The critical slope corresponding to the discharge $Q = 4000$ cfs is determined (see procedure 6).

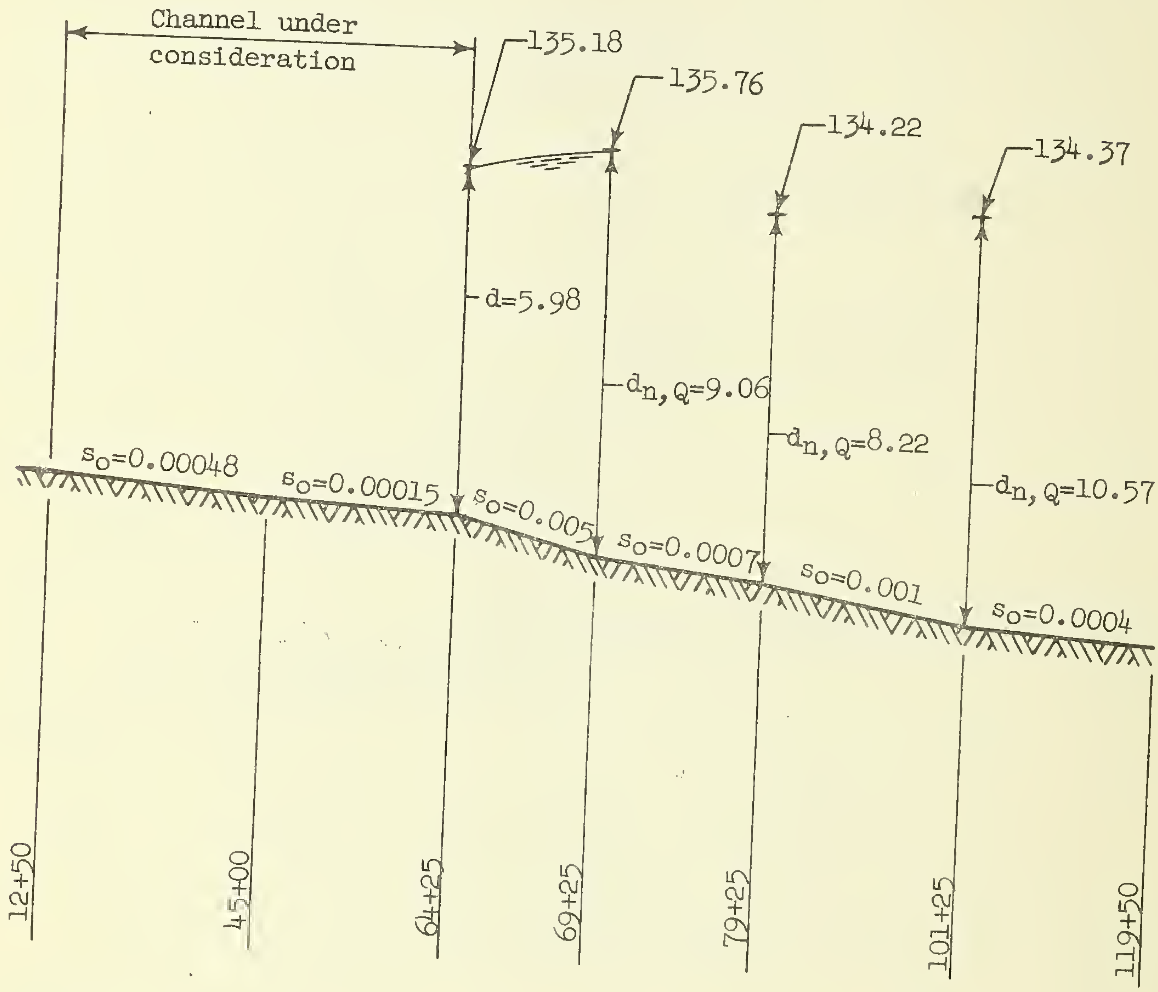
$$s_{c,Q} = 0.00180 \text{ ft/ft} > s_o = 0.0007$$

Thus, there is a mild slope downstream from the first break in grade below the downstream end of the channel under consideration (the reach between 69 + 25 and 79 + 25).

The selected channel reach $(\ell_2 - \ell_1) = ([69 + 25] - [64 + 25]) = 501$ ft contains the break in grade at station 69 + 25.

Compute $d_{n,Q}$ for the bottom slope $s_o = 0.0007$ (see procedure 4) $d_{n,Q} = 9.06$ ft. Compute the water surface profile upstream from station 69 + 26 using the starting depth $d_{n,Q} = 9.06$ ft to a depth d = 5.98 ft at station 64 + 25 (see procedure 17).

Compute $d_{n,Q}$ for $s_o = 0.001$ and $s_o = 0.0004$ for stations 79 + 26 and 101 + 26, obtain 8.22 ft and 10.57 ft, respectively. These depths give the elevation at these stations as shown in the figure below.



The downstream conditions from station 101 + 25 indicates that the depth of flow at station 101 + 25 is at least $d_{n,Q} = 10.57$ ft. Thus, the tailwater conditions would influence the depth of flow at station 79 + 25. Compute the water surface profile upstream from station 101 + 26 starting at the depth $d_{n,Q} = 10.57$ ft. This gives the starting depth d = 6.02 ft at station 64 + 25.

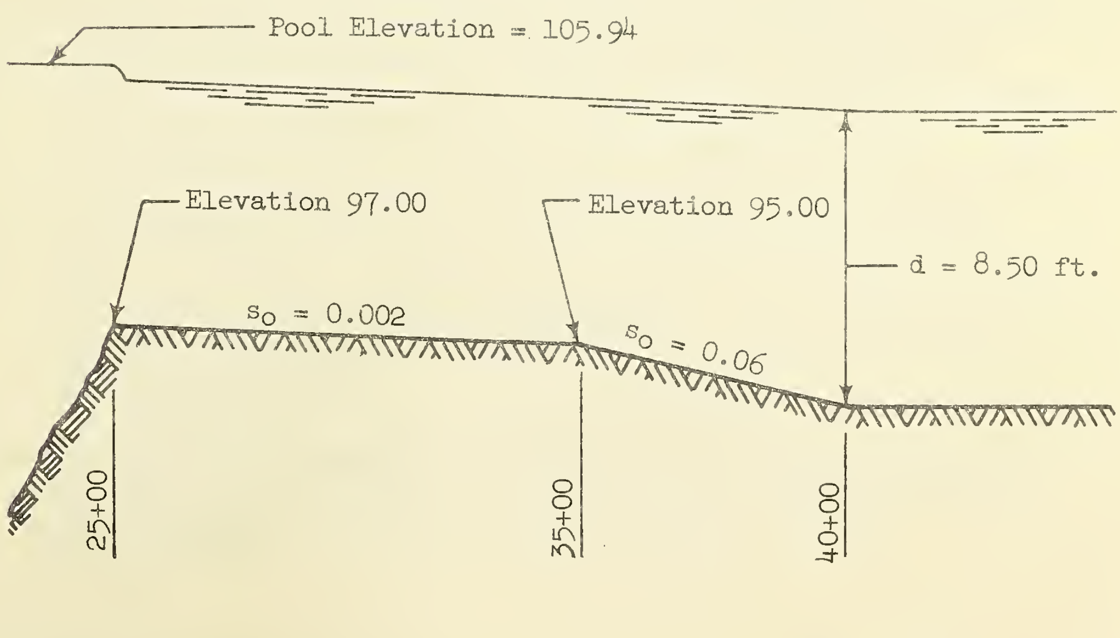
The error (6.02 - 5.98) is less than the tolerable error of 0.05 ft, and the starting depth at station 64 + 25 is taken as 6.02 ft.

Compute the profile to the upstream end of the channel under consideration. A tabulation of reach lengths $(\ell_2-\ell_1)$ and corresponding depths are given below for each step. Since there are no steep slopes upstream, the profile for the channel under consideration is complete.

$n^2(\ell_2-\ell_1)$	$(\ell_2-\ell_1)$	$\Sigma(\ell_2-\ell_1)$	Depth	Station
0	0	0	6.02	64 + 25
0.05	125	125	6.90	63 + 00
0.10	250	375	7.60	60 + 50
0.10	250	625	8.02	58 + 00
0.10	250	875	8.33	55 + 50
0.10	250	1125	8.58	53 + 00
0.10	250	1375	8.80	50 + 50
0.10	250	1625	9.00	48 + 00
0.10	250	1875	9.15	45 + 50
0.02	50	1925	9.18	45 + 00
0.40	1000	2925	9.38	35 + 00
0.40	1000	3925	9.52	25 + 00
0.40	1000	4925	9.63	15 + 00
0.10	250	5175	9.64	12 + 50

EXAMPLE 18b

Given: (1) Trapezoidal channel, z = 2
(2) b = 20.0 ft
(3) n = 0.04
(4) Tailwater depth at station 40 + 00 = 8.50 ft
(5) Pool elevation at upstream end of channel under consideration is 105.94
(6) The profile of the channel under consideration is shown in the following figure.



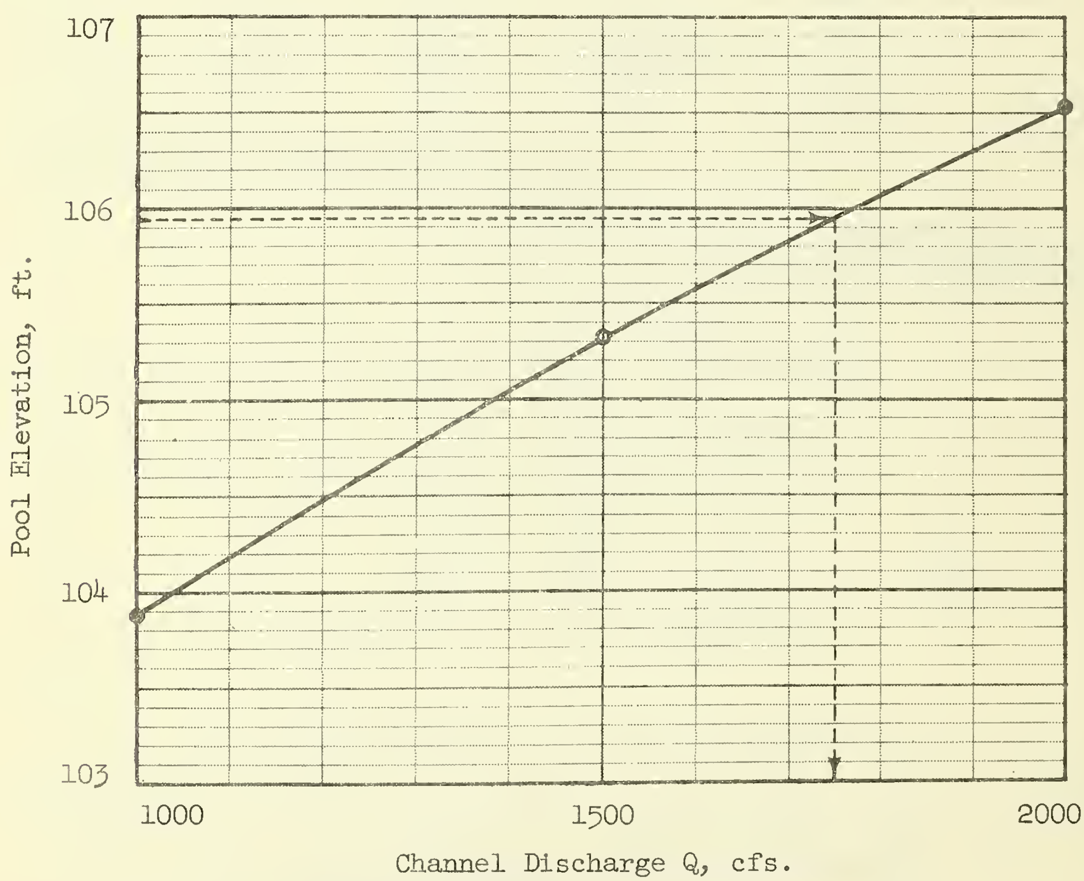
Determine: The steady discharge Q in the channel under consideration for the given tailwater depth d and headwater condition.

Solution: Select several discharges such that the computed pool elevations for two of the selected discharges will bracket the known pool elevation.

Select, in turn, Q = 1000 cfs, Q = 1500 cfs, and Q = 2000 cfs. Start all water surface profile computations at station 40 + 00 with d = 8.50 ft, and compute the profiles for the selected discharges to station 25 + 00. (See sheet 4 of ES-142).

Compute the pool elevation by adding the depth d and the velocity head $\left[\frac{v^2}{2g}\right]$ (corresponding to the selected discharges) to the channel bottom elevation 97.00 ft at station 25 + 00.

Plot the following rating curve (pool elevation vs. discharge Q).

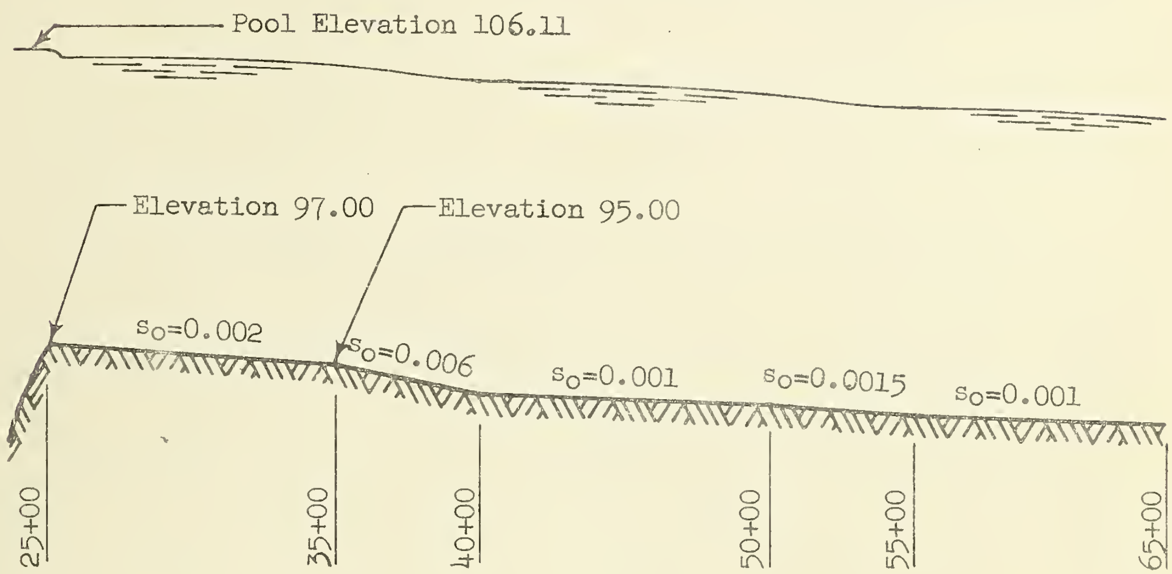


Rating curve of pool at exit station 25 + 00

Read channel discharge Q = 1750 cfs corresponding to the given pool elevation 105.94

EXAMPLE 18c

Given:
(1) Trapezoidal channel, z = 2
(2) b = 20 ft
(3) n = 0.04
(4) Pool elevation at upstream end of channel under consideration is 106.11
(5) The profile of the channel under consideration is shown in the following figure.
(6) Tolerable error = 0.10 ft



Determine: The discharge Q of the channel and the channel depth of flow at station 25 + 00.

Solution: Select the discharge Q = 1000 cfs. Since there is no starting depth given, the computational procedure will begin as given on sheet 4 of ES-142. Begin the water surface profile computation at station 40 + 00 for Q = 1000 cfs with an assumed depth $d_{n,Q}$ = 7.97 ft (where s_o = 0.001). Compute the water surface profile upstream to station 25 + 00 where d = 6.50 (See ES-142, sheet 4.) Begin a new water surface profile computation at station 55 + 00 for a Q = 1000 cfs with an assumed depth $d_{n,Q}$ = 7.97 ft (where s_o = 0.001). Compute the water surface profile upstream to station 25 + 00 where d = 6.49 The difference of the two computed profile depths = 6.50 - 6.49 = 0.01 is less than the tolerable error of 0.10 ft. The last computed profile is acceptable.

Compute the pool elevation by adding the depth d and the velocity head $\left[\frac{v^2}{2g}\right]$ (corresponding to the discharge Q = 1000 cfs) to the channel bottom elevation 97.00 ft at station 25 + 00.

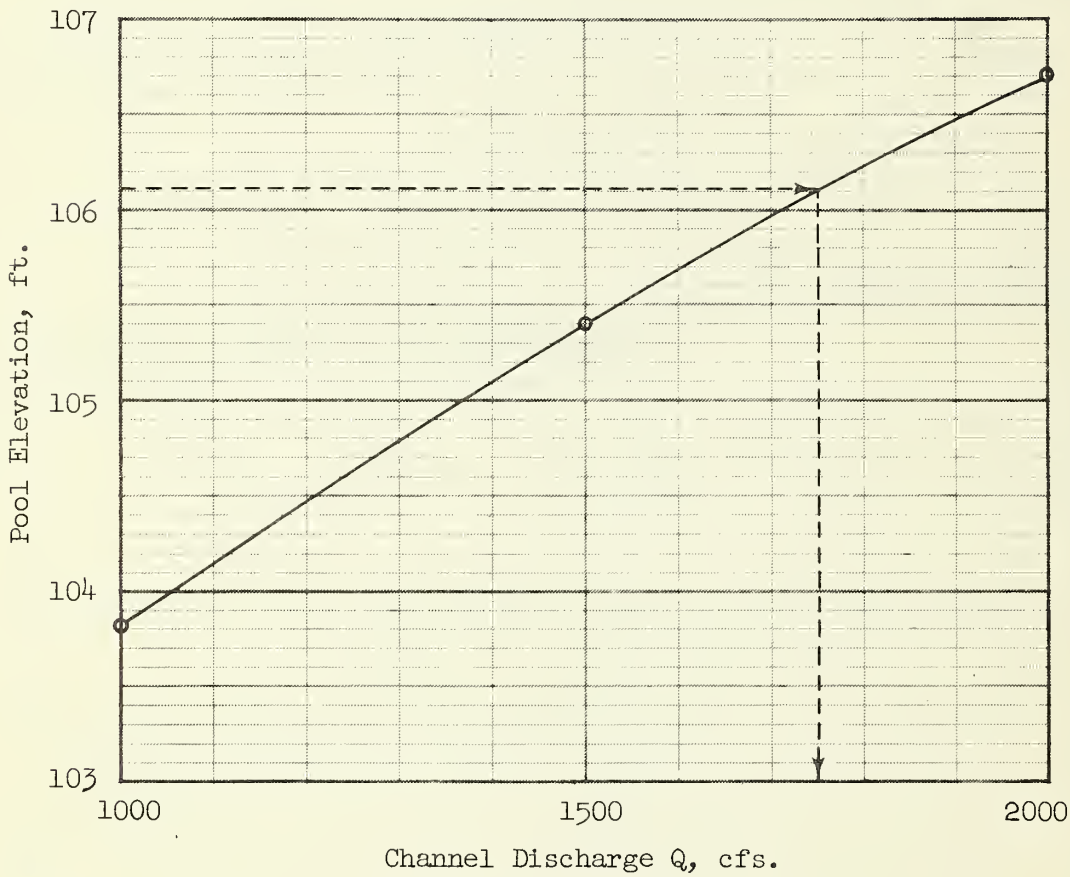
That is,

$$\text{pool elevation} = 6.82 + 97.00 = 103.82 \text{ ft}$$

Since the computed pool elevation for the selected discharge is less than the given pool elevation, greater discharges are selected and computed until the computed pool elevations bracket the known pool elevation.

Compute pool elevations for the discharges Q = 1500 cfs and Q = 2000 cfs, following the same procedure as given above.

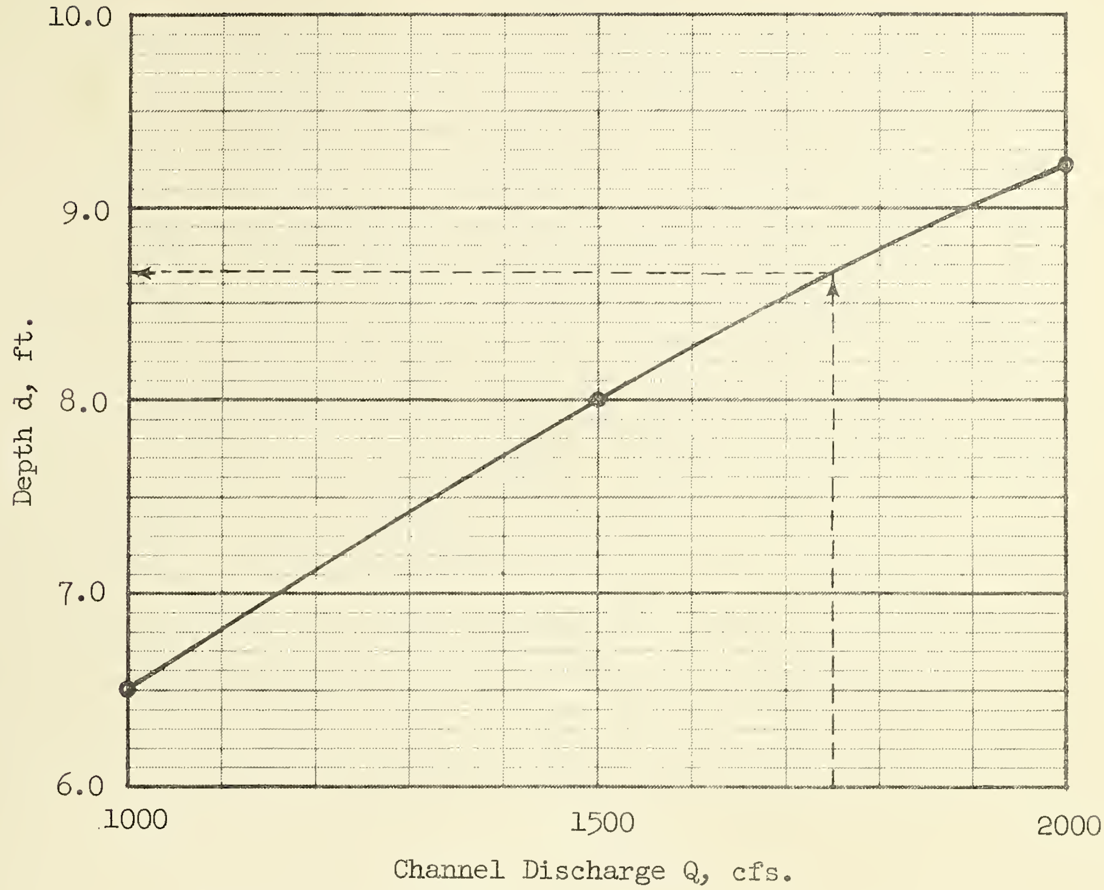
Plot the rating curve (pool elevation vs. discharge Q) as shown in the following figure.



Rating curve of pool at exit station 25 + 00

Read the discharge Q = 1750 cfs corresponding to the given pool elevation.

Plot the rating curve (depth d vs. discharge Q) for the channel at station 25 + 00.



Rating at channel entrance station 25 + 00

Read the depth d = 8.65 corresponding to the determined discharge Q = 1750 cfs.